Jets at strong coupling

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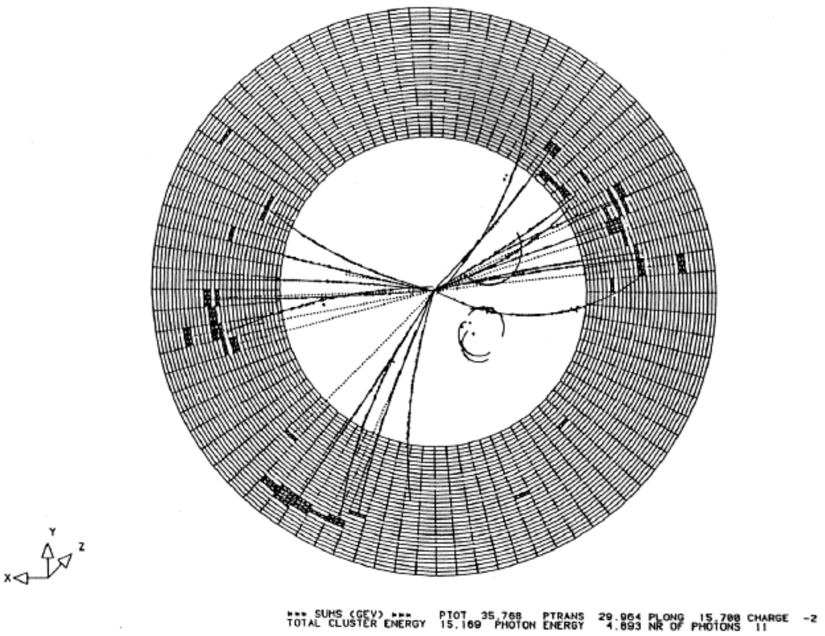
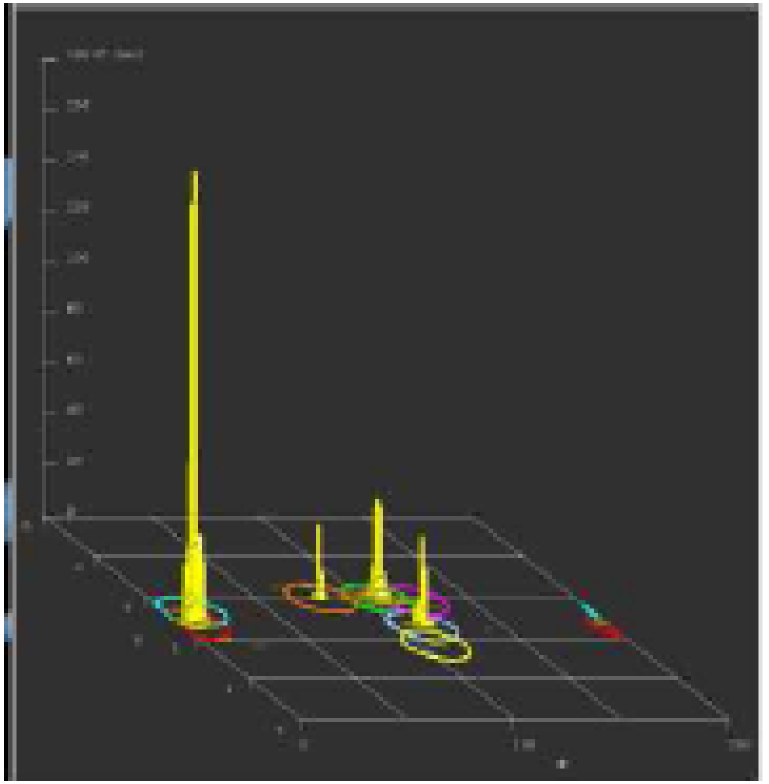
Jets in QCD

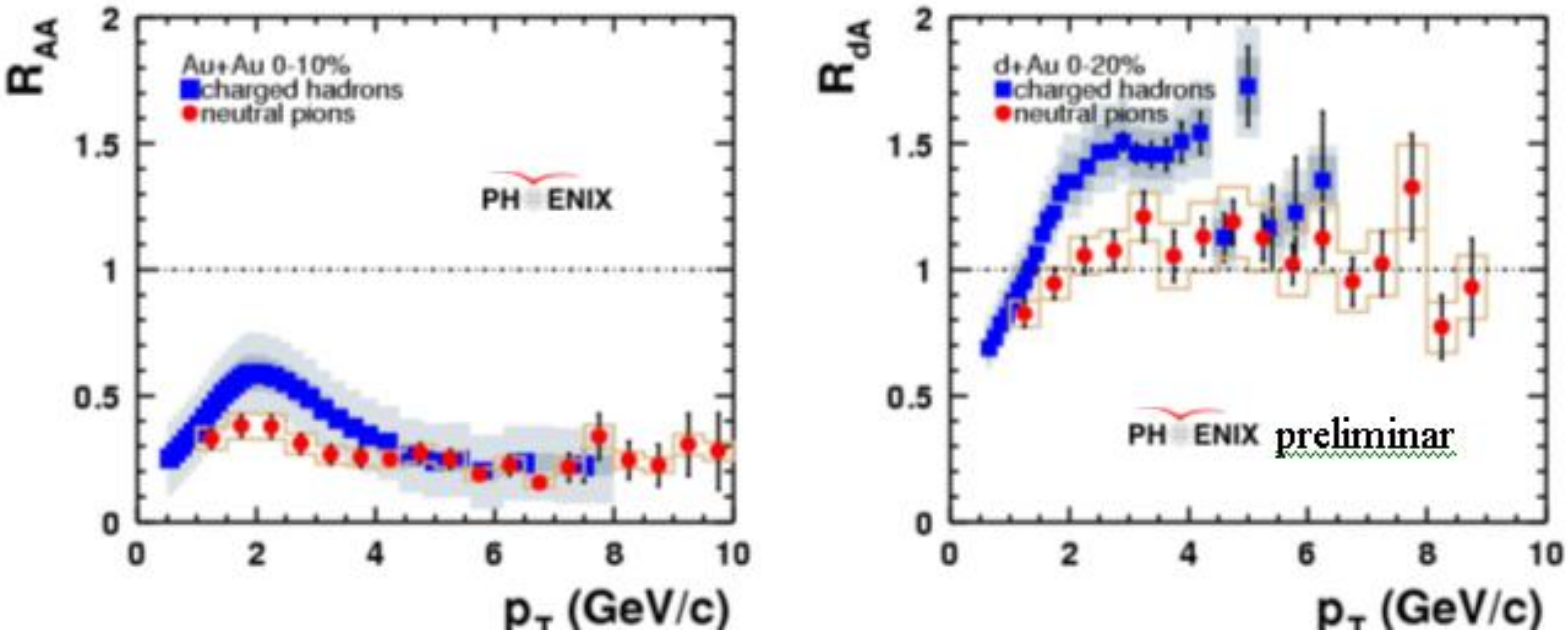




In e+e- annihilation, the most stringent tests of pQCD have been done.

High pt jets at the LHC could be an important discovery channel of BSM

Jets in heavy-ion collisions



Observed jet quenching of high-pt hadrons seems to be stronger than pQCD predictions.

Important probe of the ``sQGP”—strongly coupled quark gluon plasma

The stopping distance at strong coupling

Massless $Q^2 = 0$, or more generally, $ET^2 \gg Q^3$

$$t \propto E^{1/3}$$

Gubser, Gulotta, Pufu, Rocha (gluon)
YH, Iancu, Mueller (virtual photon)
Chesler, Jensen, Karch, Yaffe (quark)

Moderate energy $Q^2 T < ET^2 < Q^3$

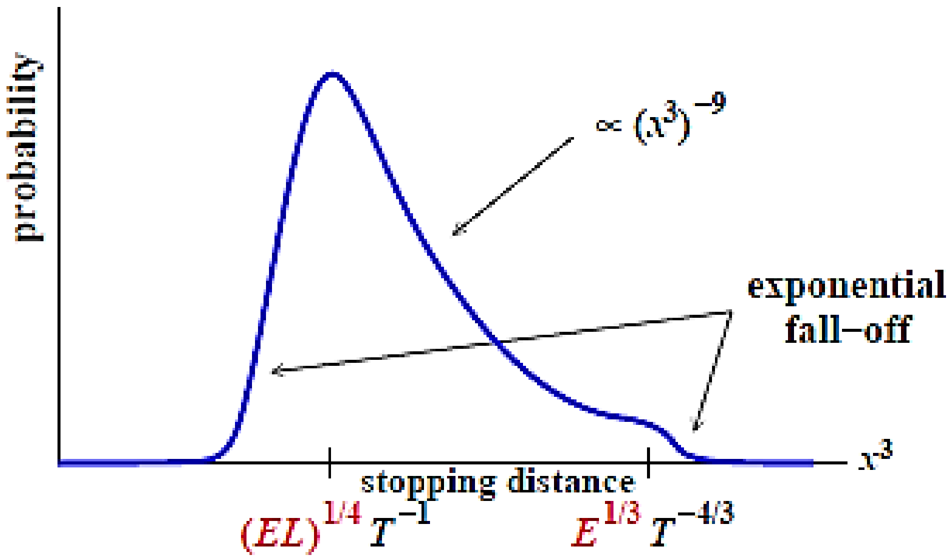
$$t \propto E^{1/4}$$

YH, Iancu, Mueller,
Arnold, Vaman

c.f. $t \propto E^{1/2}$ at weak coupling

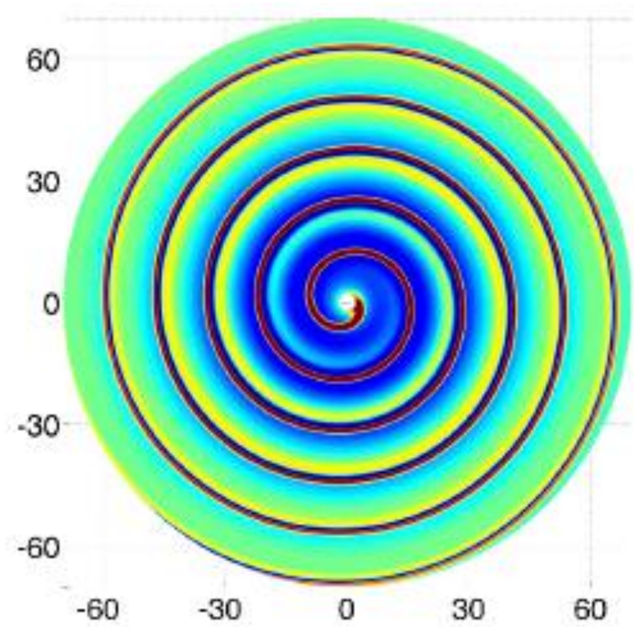
Wavepacket analysis

Arnold, Vaman



Synchrotron radiation

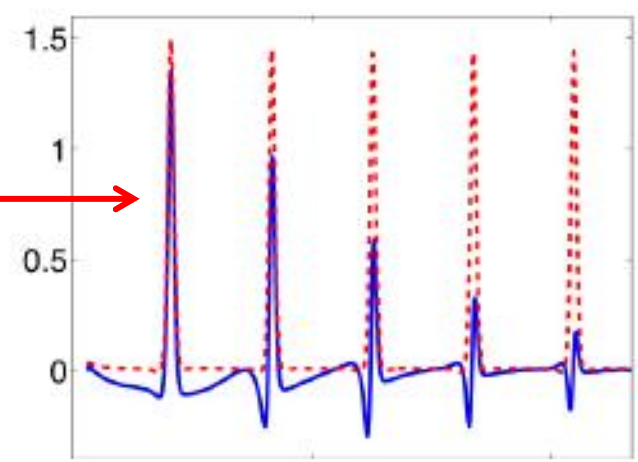
Chesler, Ho, Rajagopal



Very small width !

…like in classical electrodynamics.

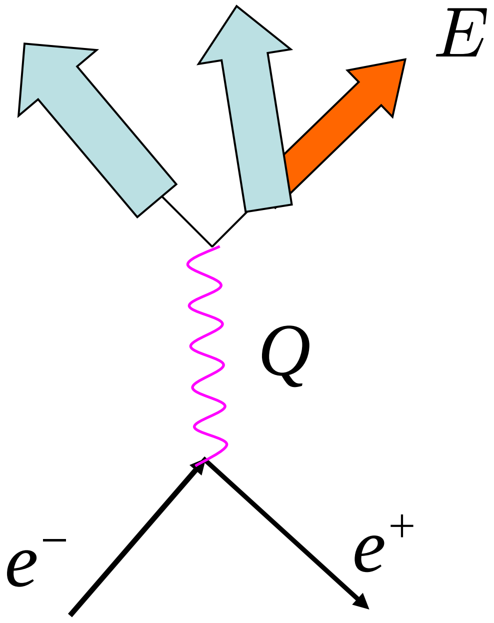
Conflict with quantum mechanics?



YH, Iancu, Mueller, Triantafyllopoulos

Jets in QCD:
The inclusive spectrum

Cross section to produce one hadron with energy E plus anything else



$$\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx} = D_T(x, Q^2) \qquad x = \frac{2E}{Q}$$

Fragmentation function

Feynman-x

Counts how many hadrons are there inside a quark.

DGLAP equation

$$\frac{\partial}{\partial \ln Q^2} D_T(x, Q^2) = \int_x^1 \frac{dz}{z} P_T(z) D_T\left(\frac{x}{z}, Q^2\right)$$

↓

Mellin transform in x

$$D_T(j, Q^2) = \int_0^1 dx \; x^{j-1} D_T(x, Q^2)$$

$$\frac{\partial}{\partial \ln Q^2} D_T(j, Q^2) = \gamma_T(j) D_T(j, Q^2)$$

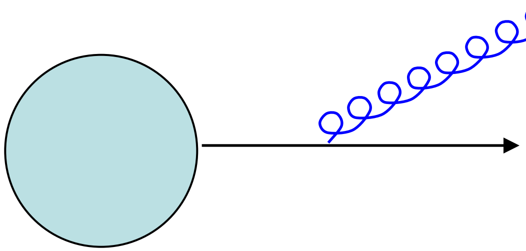
Timelike anomalous dimension

First moment gives the average multiplicity

$$\langle n \rangle = D_T(1, Q^2) \propto Q^{2\gamma_T(1)}$$

Timelike anomalous dimension

Lowest order perturbation



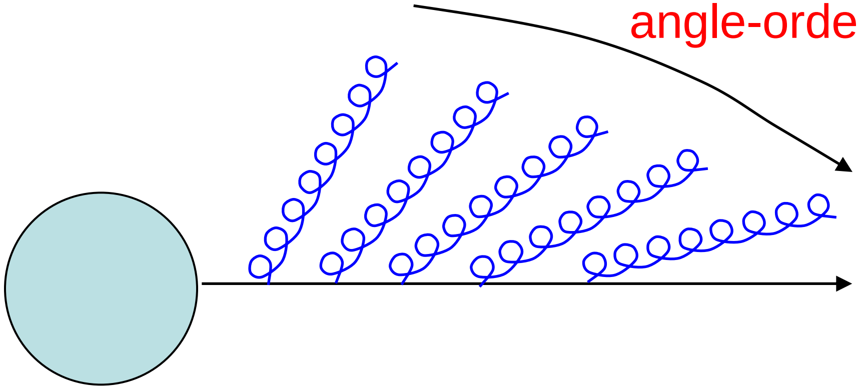
$$\sim \frac{1}{x}$$

Soft singularity

$$\longleftrightarrow \quad \gamma_T(j) \sim \frac{\alpha_s}{j-1}$$

$$\gamma_T(1) = \infty \quad !!$$ Nonsense !

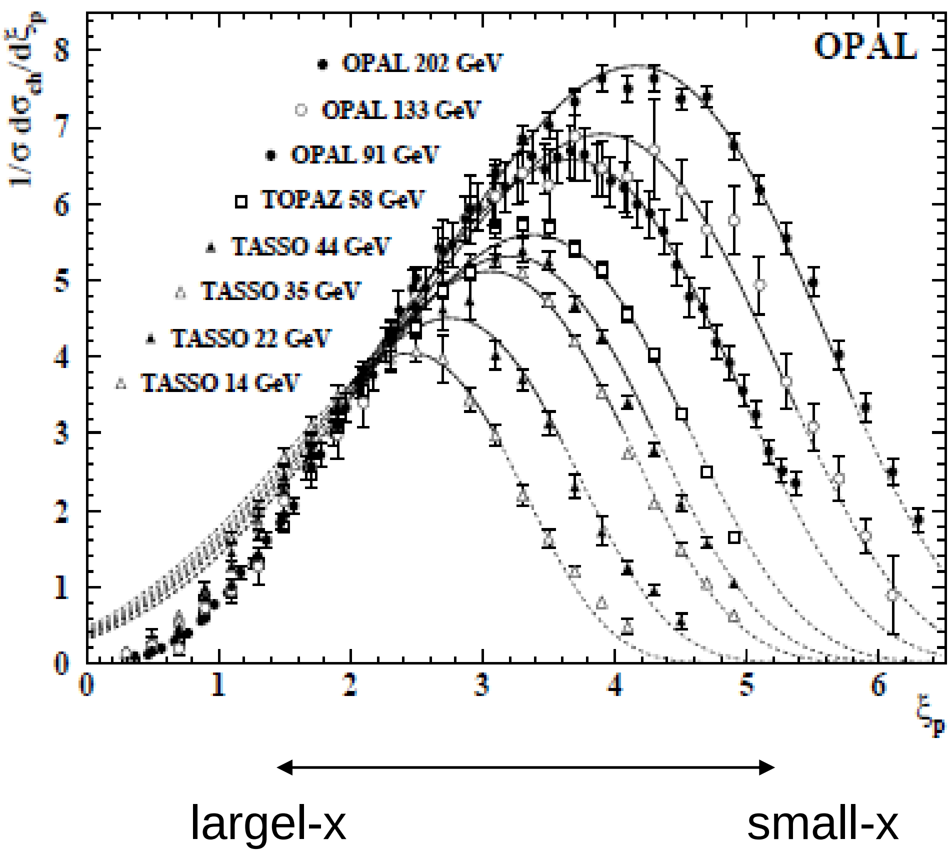
Resummation



$$\gamma_T(j) = \frac{1}{4}\left[\sqrt{(j-1)^2 + \frac{8N\alpha_s}{\pi}} - (j-1)\right]$$

$$\gamma_T(1) = \sqrt{\frac{N\alpha_s}{2\pi}}$$ Mueller (1981)

Inclusive spectrum



$$\frac{x}{\sigma}\frac{d\sigma}{dx} \propto xD_T(x,Q^2)$$

"hump-backed" distribution peaked at

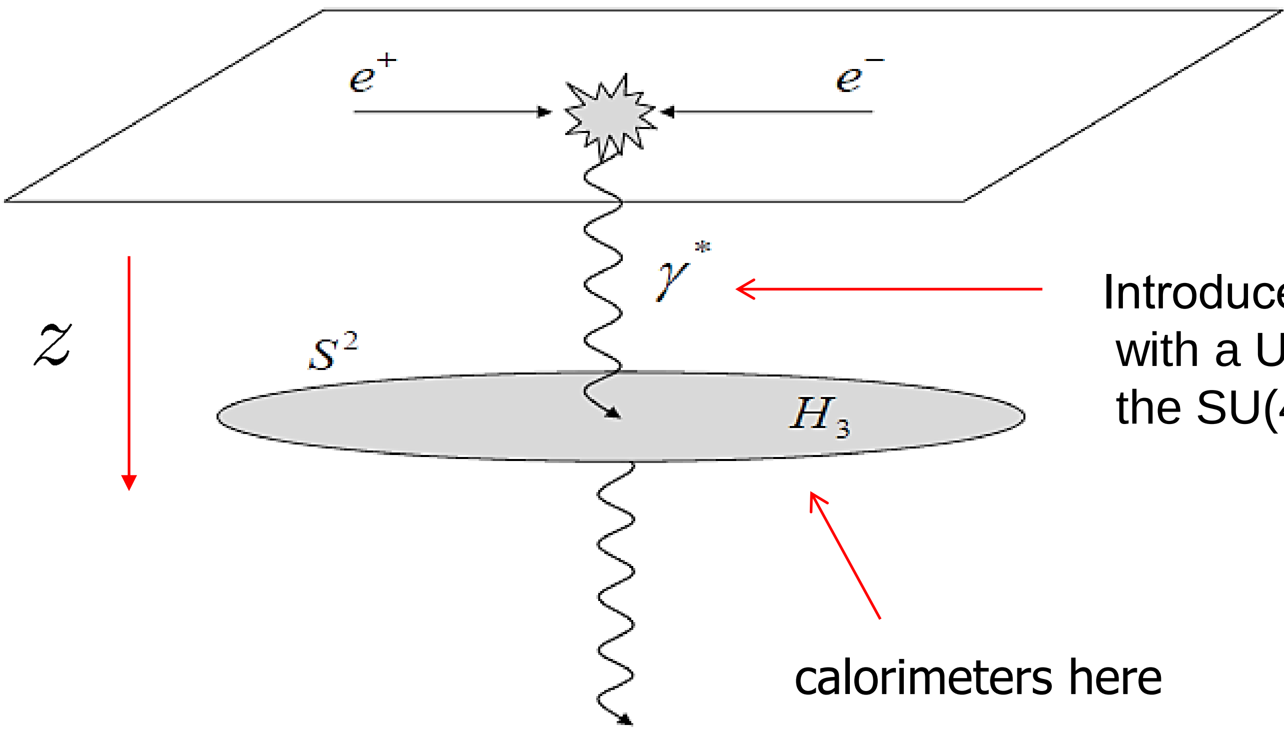
$$\frac{1}{x} \sim \sqrt{\frac{Q}{\Lambda}}$$

Double logs + QCD coherence. Structure of jets well understood in pQCD.

e+e- annihilation in N=4 SYM at strong coupling

AdS metric in the Poincare coordinates

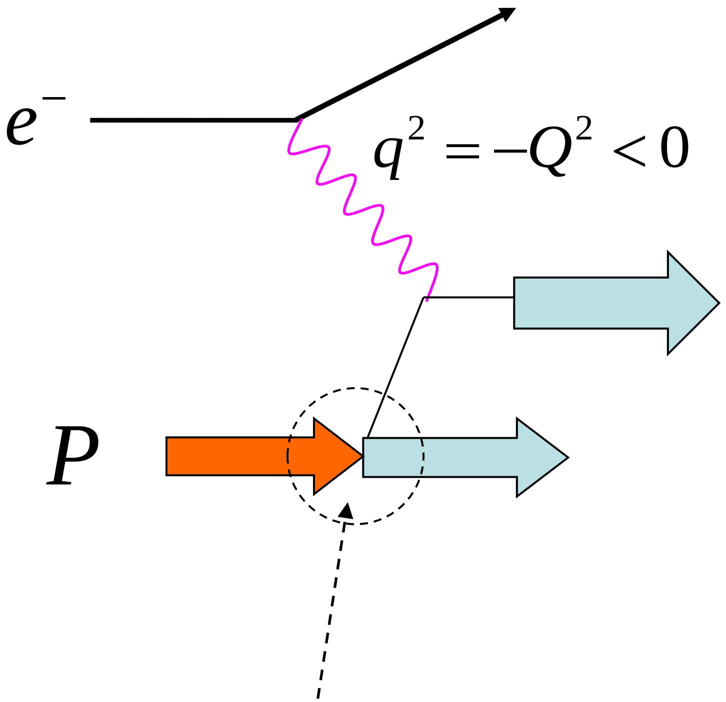
$$ds^2 = \frac{dx^\mu dx_\mu + dz^2}{z^2}$$



Introduce a “photon” associated with a U(1) subgroup of the SU(4) R-symmetry.

calorimeters here

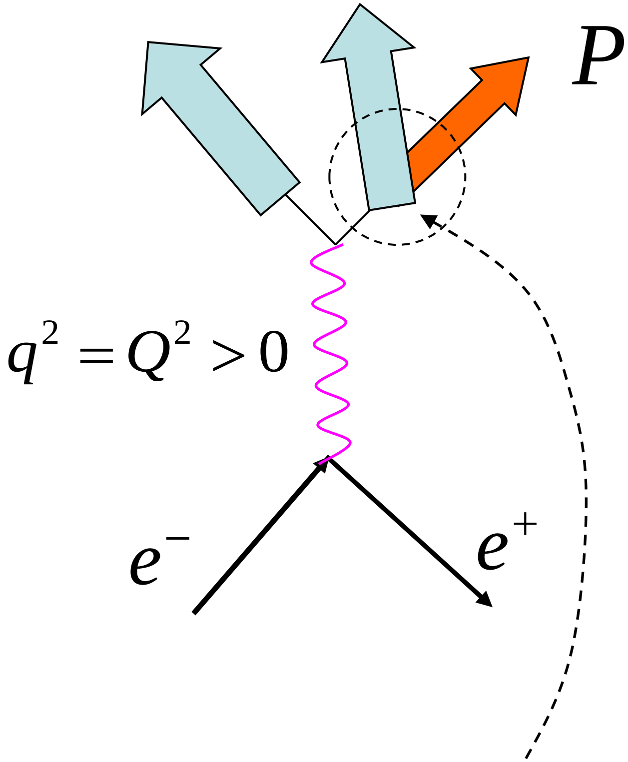
DIS vs. e+e- : crossing symmetry



Parton distribution function

$$D_S(x_B, Q^2)$$

Bjorken variable $\quad x_B \equiv \dfrac{Q^2}{2P \cdot q}$



Fragmentation function

$$D_T(x_F, Q^2)$$

Feynman variable $\quad x_F \equiv \dfrac{2P \cdot q}{Q^2}$

Gribov-Lipatov reciprocity

DGLAP equation
$$\frac{\partial}{\partial \ln Q^2} D_{S/T}(j,Q^2) = \gamma_{S/T}(j) D_{S/T}(j,Q^2)$$

The two anomalous dimensions derive from a single function

$$\gamma_S(j) = f(j - \gamma_S(j))$$
$$\gamma_T(j) = f(j + \gamma_T(j))$$

Dokshitzer, Marchesini, Salam

Nontrivial check up to three loops (!) in QCD Mitov, Moch, Vogt

Multiplicity at strong coupling

$$\gamma_S(j)=\frac{j}{2}-\frac{1}{2}\sqrt{2\sqrt{\lambda}(j-j_0)} \quad \overset{\text{crossing}}{\longleftrightarrow} \quad \gamma_T(j)=-\frac{1}{2}\left(j-j_0-\frac{j^2}{2\sqrt{\lambda}}\right)$$

Lipatov et al.
Brower et al.

$\lambda = g^2 N_c$

$$n(Q)\propto (Q/\Lambda)^{2\gamma_T(1)}=(Q/\Lambda)^{1-3/2\sqrt{\lambda}}$$

YH, Matsuo

c.f. in perturbation theory, $n(Q)\propto Q^{\sqrt{\frac{\lambda}{2\pi^2}}}$

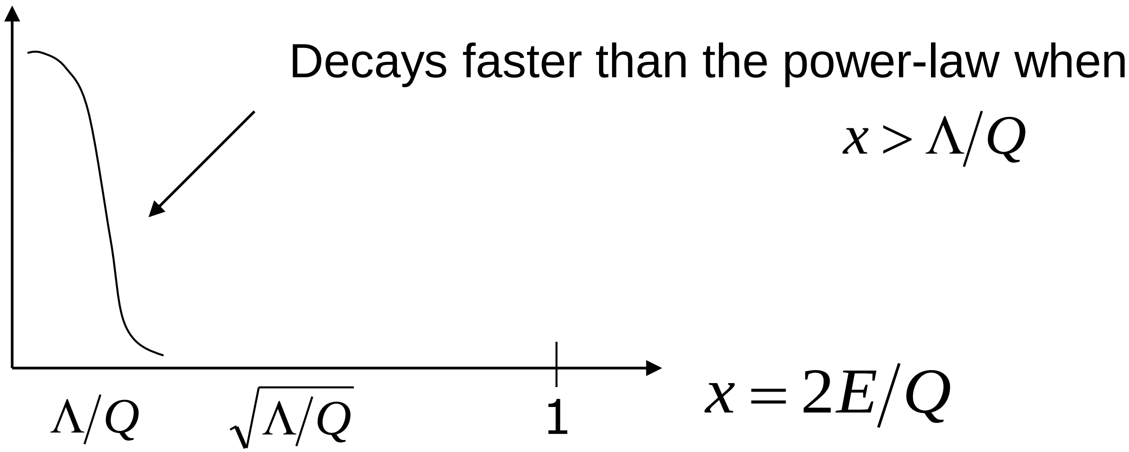
c.f. heuristic argument $n(Q)\propto Q$ YH, Iancu, Mueller

Jets at strong coupling?

$$\gamma_T(j) \approx 1 - \frac{j}{2}$$ in the supergravity limit $\lambda \to \infty$

The inclusive distribution is peaked at the kinematic lower limit

$$D_T(x, Q^2) = Q^2 F\left(\frac{Qx}{\Lambda}\right)$$



At strong coupling, branching is so fast and complete.
There are no “partons” at large-x.

Energy correlation function

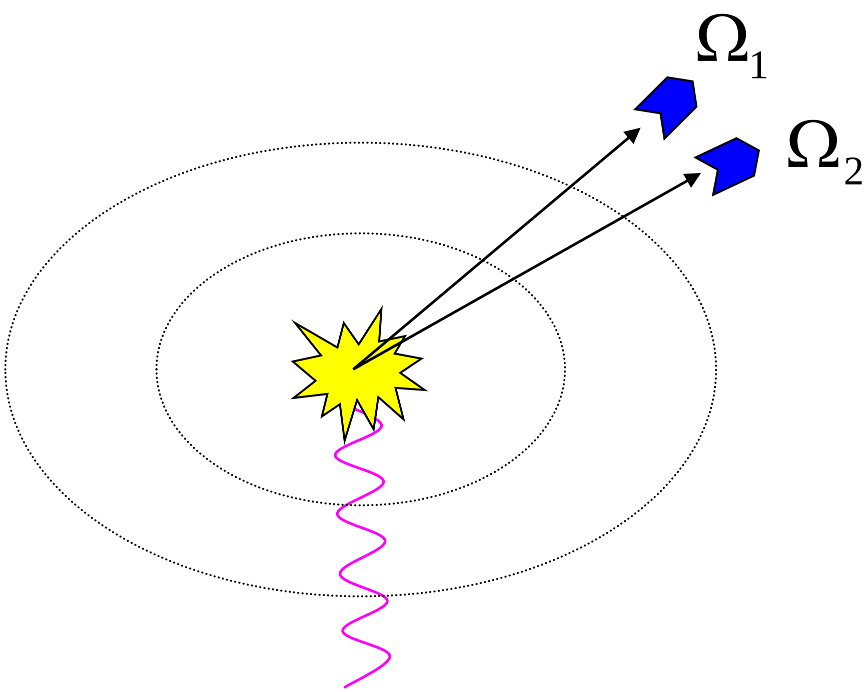
Hofman, Maldacena

Energy distribution is spherical for any λ
Correlations disappear as $\lambda \to \infty$

$$\langle \mathcal{E}(\Omega_1)\mathcal{E}(\Omega_2)\rangle \sim \frac{1}{|\theta_{12}|^{2+2\gamma_S(3)}}$$

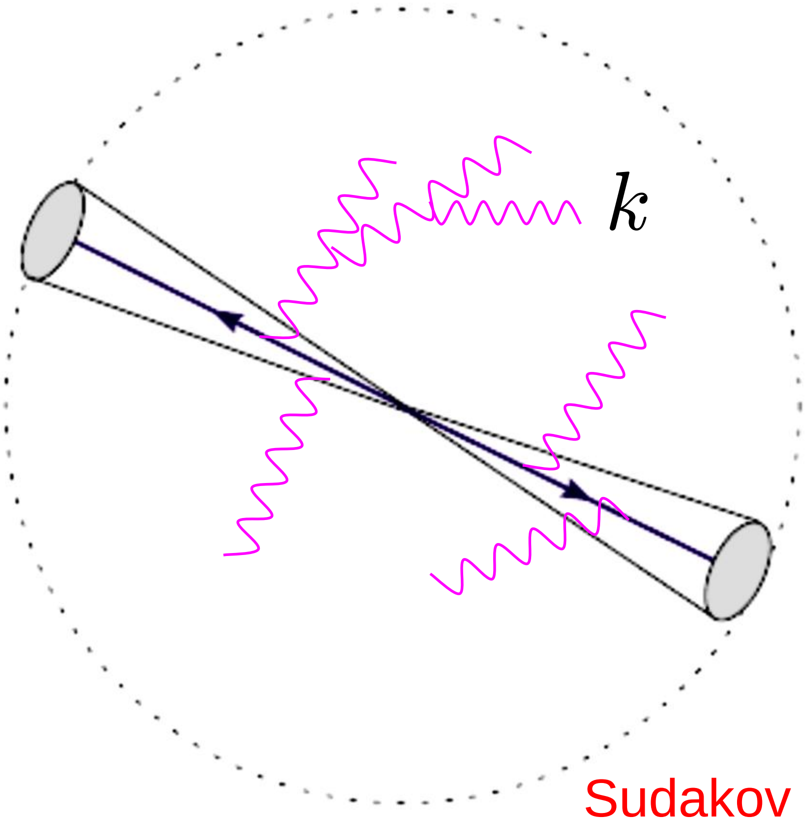
$$\gamma_S(3) = O(\lambda) << 1$$ weak coupling

$$= -\lambda^{1/4}/\sqrt{2}$$ strong coupling



All the Q/Λ particles have the minimal four momentum $\sim \Lambda$ and are spherically emitted. There are no jets at strong coupling !

Away-from-jets region



Gluons emitted at large angle, insensitive to the collinear singularity

Resum only the soft logarithms

$$\left(\alpha_s \ln \frac{E_{jet}}{k}\right)^n$$

There are two types of logarithms.

Sudakov logs. (emission from primary partons)

Kidonakis, Oderda, Sterman

Non-global logs. (emission from secondary gluons)

Dasgupta, Salam

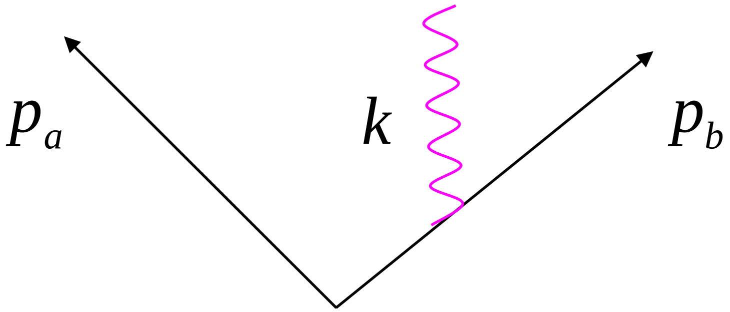
Marchesini-Mueller equation (2003)

Evolution of the gluon distribution between jets.

$$\partial_Y n(\theta_{ab},\theta_{cd},Y) = \bar{\alpha}_s \int \frac{d^2\Omega_k}{4\pi} \frac{1-\cos\theta_{ab}}{(1-\cos\theta_{ak})(1-\cos\theta_{bk})} \times \left(n(\theta_{ak},\theta_{cd},Y) + n(\theta_{bk},\theta_{cd},Y) - n(\theta_{ab},\theta_{cd},Y) \right).$$

Probability of gluon emission

$$Y = \ln \frac{E_{jet}}{k}$$

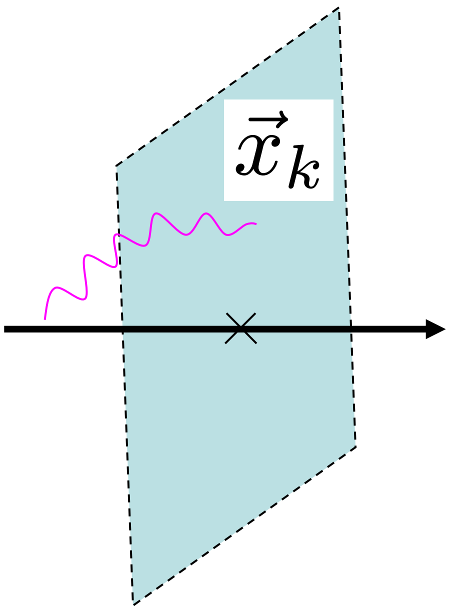


BFKL equation

Evolution of the gluons in the transverse plane

$$\partial_Y n(x_{ab}, x_{cd}, Y) = \bar{\alpha}_s \int \frac{d^2\vec{x}_k}{2\pi} \frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2(\vec{x}_{bk})^2} \times \left(n(x_{ak}, x_{cd}, Y) + n(x_{bk}, x_{cd}, Y) - n(x_{ab}, x_{cd}, Y)\right)$$

$$Y = \ln \frac{E}{k}$$



Exact solution known
thanks to 2D conformal symmetry.

$$n \sim e^{4\alpha_s \ln 2 Y} = \left(\frac{1}{x}\right)^{4\alpha_s \ln 2}$$

Two Poincaré coordinates

AdS_5 as a hypersurface in 6D

②

$$W_{-1}^2 + W_0^2 - W_1^2 - W_2^2 - W_3^2 - W_4^2 = R^2$$

①

AdS_5 in Poincare coordinates

$$ds^2 = \frac{dx^\mu dx_\mu + dz^2}{z^2}$$

Introduce two Poincaré coordinate systems Cornalba

Our universe

Poincaré 1 : $W_{-1} + W_4 = \frac{1}{z}, \quad W_\mu = \frac{x^\mu}{z}. \quad (\mu = 0, 1, 2, 3)$

Poincaré 2 : $W_0 + W_3 = \frac{1}{y_5} \quad W_{-1} = -\frac{y^0}{y_5}, \quad W_4 = -\frac{y^3}{y_5}, \quad W_{1,2} = \frac{y^{1,2}}{y_5}$

Shock wave picture of e+e- annihilation

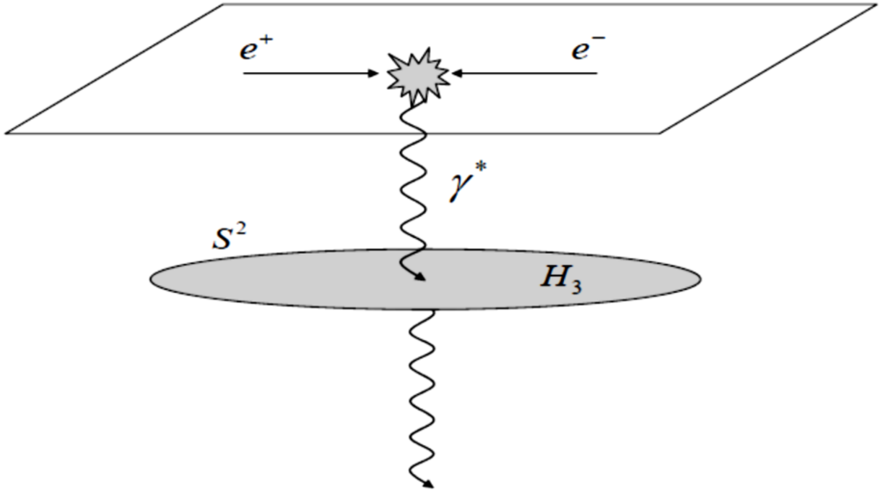
Y.H.

Angular distribution of energy $\mathcal{E}(\Omega) \equiv \lim_{r\to\infty} r^2 \int_0^\infty dx^0\, n_i T^{0i}(x^0, r\vec{n})$

The sphere Ω can be mapped onto the transverse plane $\vec{y}_T$ of Poincare 2 via the stereographic projection

Treat the photon as a shock wave in Poincare 2

$$T_{--} = q^+ \delta(y_5 - 1)\delta^{(2)}(\vec{y}_T)\delta(y^-)$$



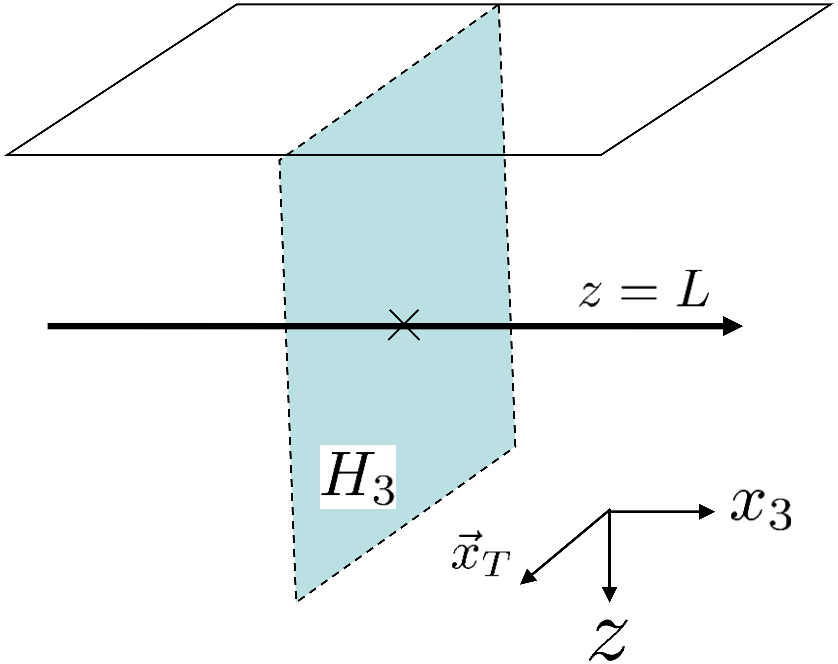
Boundary energy from the holographic renormalization

$$\langle \mathcal{E}(\Omega)\rangle = \frac{Q}{4\pi}$$

Hofman, Maldacena

Shock wave picture of a high energy "hadron"

A color singlet state lives in the bulk.
At high energy, it is a shock wave
in Poincare 1.



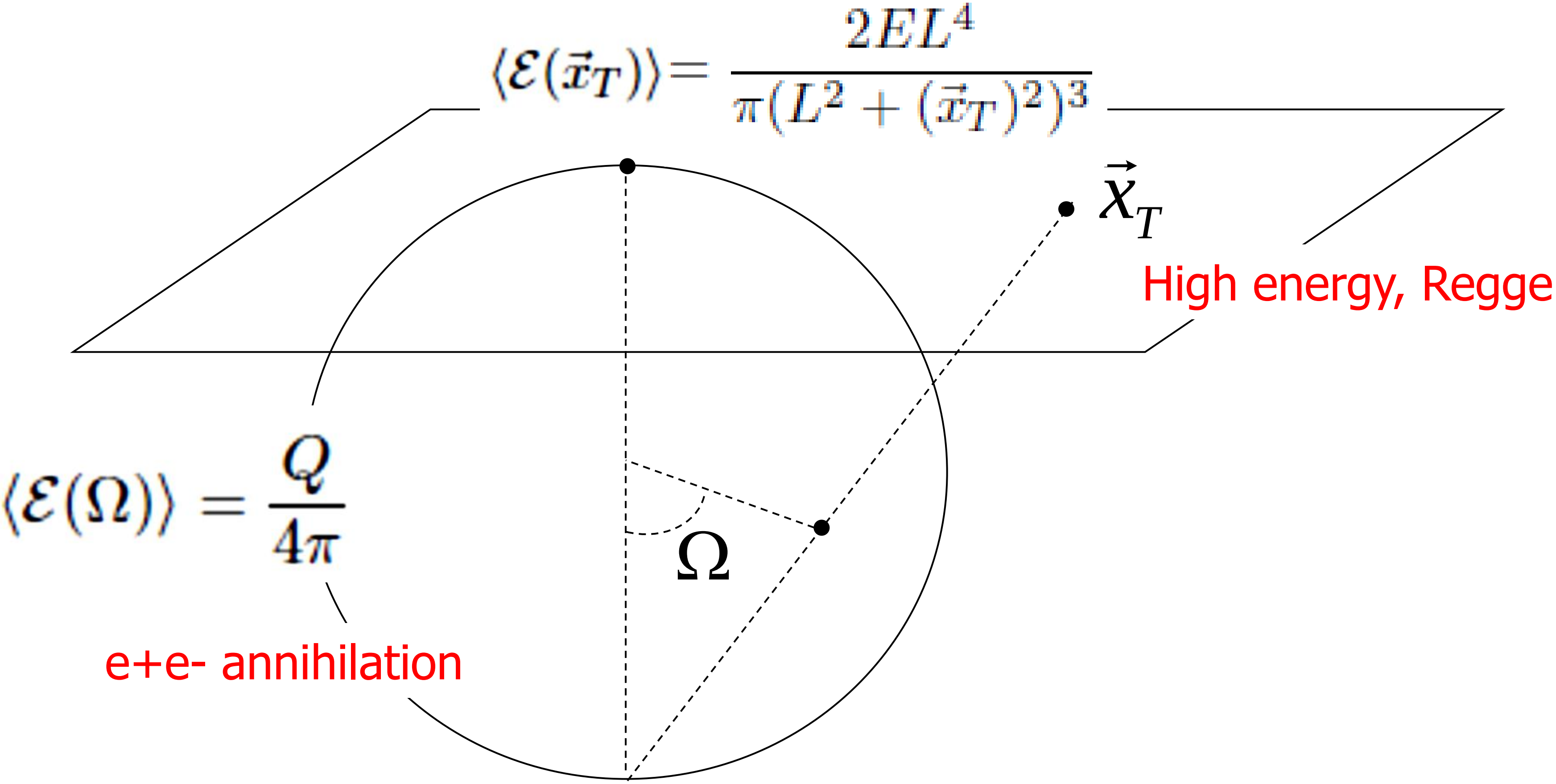
$$T^{++} = z^7 p^+ \delta(z-L)\delta^{(2)}(\vec{x}_T)\delta(x^-)$$

Energy distribution on the boundary transverse plane

$$\langle \mathcal{E}(\vec{x}_T)\rangle = \frac{1}{\sqrt{2}}\int_{-\infty}^{\infty} dx^- \langle T_{--}(x^+=0, x^-, \vec{x}_T)\rangle = \frac{2EL^4}{\pi(L^2+(\vec{x}_T)^2)^3}$$

Gubser, Pufu & Yarom

Exact map at strong coupling

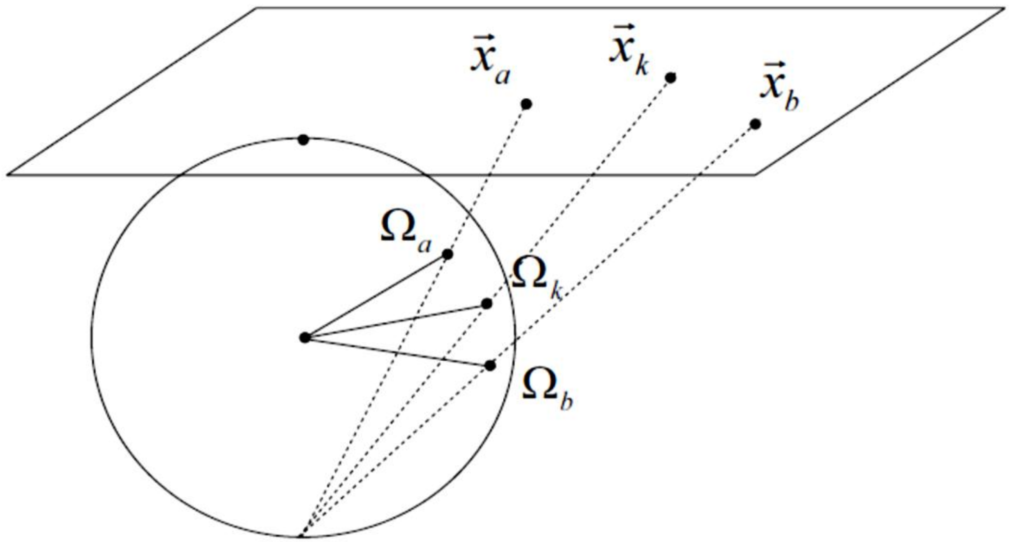


The two processes are mathematically identical.
The only difference is the choice of the coordinate systems in AdS !

Exact map at weak coupling

The same stereographic map transforms BFKL into the Marchesini-Mueller equation

$$\frac{d^2\Omega_k}{4\pi}\frac{1-\cos\theta_{ab}}{(1-\cos\theta_{ak})(1-\cos\theta_{bk})}=\frac{d^2\vec{x}_k}{2\pi}\frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2(\vec{x}_{bk})^2}$$



→ Exact solution to the Marchesini—Mueller equation

NLO evolution

The NLO BFKL in coordinate space has been calculated and shown to be conformal in N=4 SYM. Balitsky & Chirilli

NLO Marchesini-Mueller equation in N=4 Avsar, YH, Matsuo.

$$\partial_Y n_Y(\Omega_{ab}) = \bar{\alpha}_s \left(1 - \bar{\alpha}_s \frac{\pi^2}{12}\right) \int d^2\Omega_c\, K_{ab}(\Omega_c) \left[n_Y(\Omega_{ac}) + n_Y(\Omega_{cb}) - n_Y(\Omega_{ab})\right]$$

$$+\bar{\alpha}_s^2 \int d^2\Omega_c d^2\Omega_d K'_{ab}(\Omega_c, \Omega_d) n_Y(\Omega_{cd}) \,,$$

$$\begin{aligned} K'_{ab}(\Omega_c, \Omega_d) = \frac{1}{8\pi^2} \Bigg\{ & \frac{(1-\cos\theta_{ab})}{(1-\cos\theta_{ac})(1-\cos\theta_{cd})(1-\cos\theta_{db})} \\ & \times \left[\left(1 + \frac{(1-\cos\theta_{ab})(1-\cos\theta_{cd})}{(1-\cos\theta_{ac})(1-\cos\theta_{bd}) - (1-\cos\theta_{ad})(1-\cos\theta_{bc})} \right) \right. \\ & \left. \times \ln\frac{(1-\cos\theta_{ac})(1-\cos\theta_{bd})}{(1-\cos\theta_{ad})(1-\cos\theta_{bc})} + 2\ln\frac{(1-\cos\theta_{ab})(1-\cos\theta_{cd})}{(1-\cos\theta_{ad})(1-\cos\theta_{bc})} \right] \\ & + 12\pi^2 \zeta(3) \delta^{(2)}(\Omega_{ac}) \delta^{(2)}(\Omega_{bd}) \Bigg\} . \end{aligned}$$

Summary

- Heavy-ion collisions give us strong motivations to study jets at strong coupling
- Jet structure very different between weak coupling QCD and strong coupling N=4.

  (Of course!)
- Still, certain features are universal.