Non-equilibrium Phase Transitions in Holography

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Reference: S.N. Prog. Theor. Phys. 124(2010)1105.
(arXiv:1106.4105) and work in progress.

Non-equilibrium physics

- Almost all phenomena we see in our daily life are out of thermal equilibrium.
- Non-equilibrium phenomena at the vicinity of thermal equilibrium are understood in the framework of Kubo's linear response theory.
- But, it is still a challenge to describe the physics beyond the linear response region.

Our objective in this talk is to go beyond the linear response region.

Two categories of non-equilibrium states

- Time-dependent phenomena are out of equilibrium.
- Non-equilibrium steady state.

"Steady state" does not necessarily mean equilibrium.

Example:

A system with constant flow of current (along the direction of the electric field) is out of equilibrium because the heat and entropy is always created.

How to attack the non-equilibrium physics beyond the linear response?

Non-equilibrium steady state is a good place to start, comparing to the time-dependent systems.

But, how?

There is a useful framework coming from superstring theory: AdS/CFT

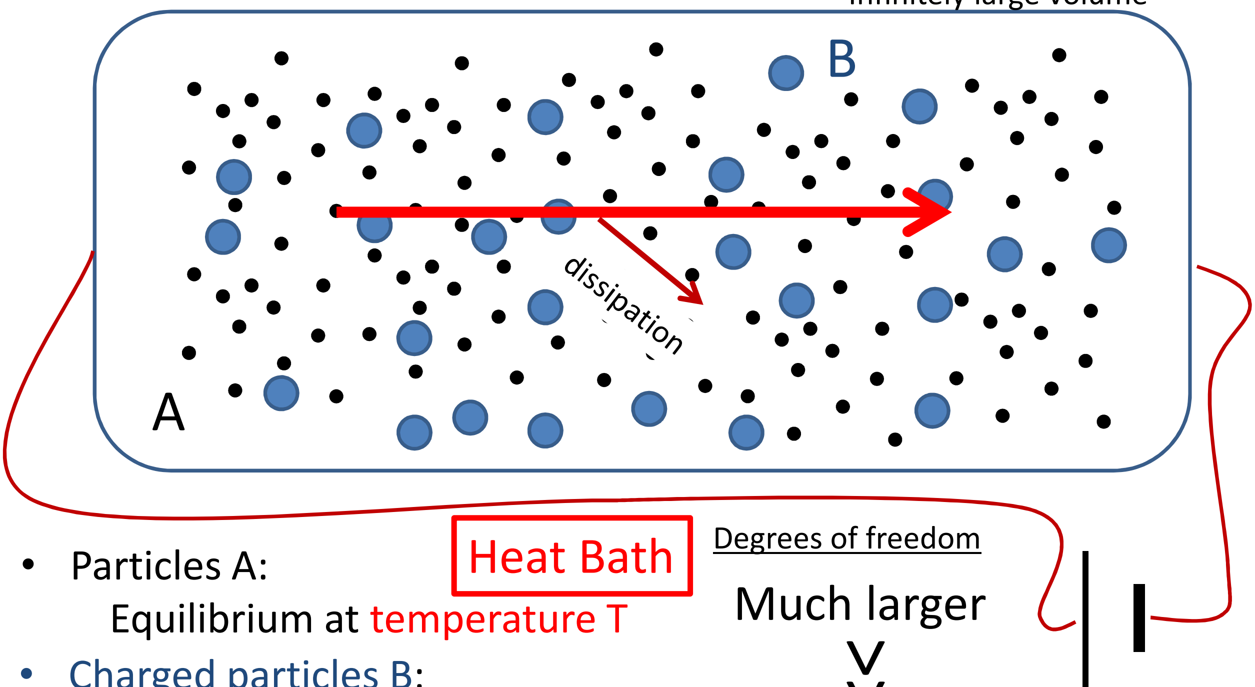
Ryogo Kubo in Keio University

Ryogo Kubo（久保亮五）1920-1995

He was a professor in Keio University, after his retirement of Yukawa Institute of Kyoto University, and University of Tokyo.

Our challenge is to go beyond him.

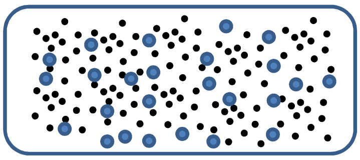
Setup: non-equilibrium steady state



- Particles A:
  Equilibrium at temperature T
- Charged particles B:
  Driven by the external force (E),
  not necessarily at thermal equilibrium.

How to apply the AdS/CFT correspondence?

AdS/CFT can deal with strongly-interacting gauge theories.



● Particle A
● Particle B

Gauge theory

gluons

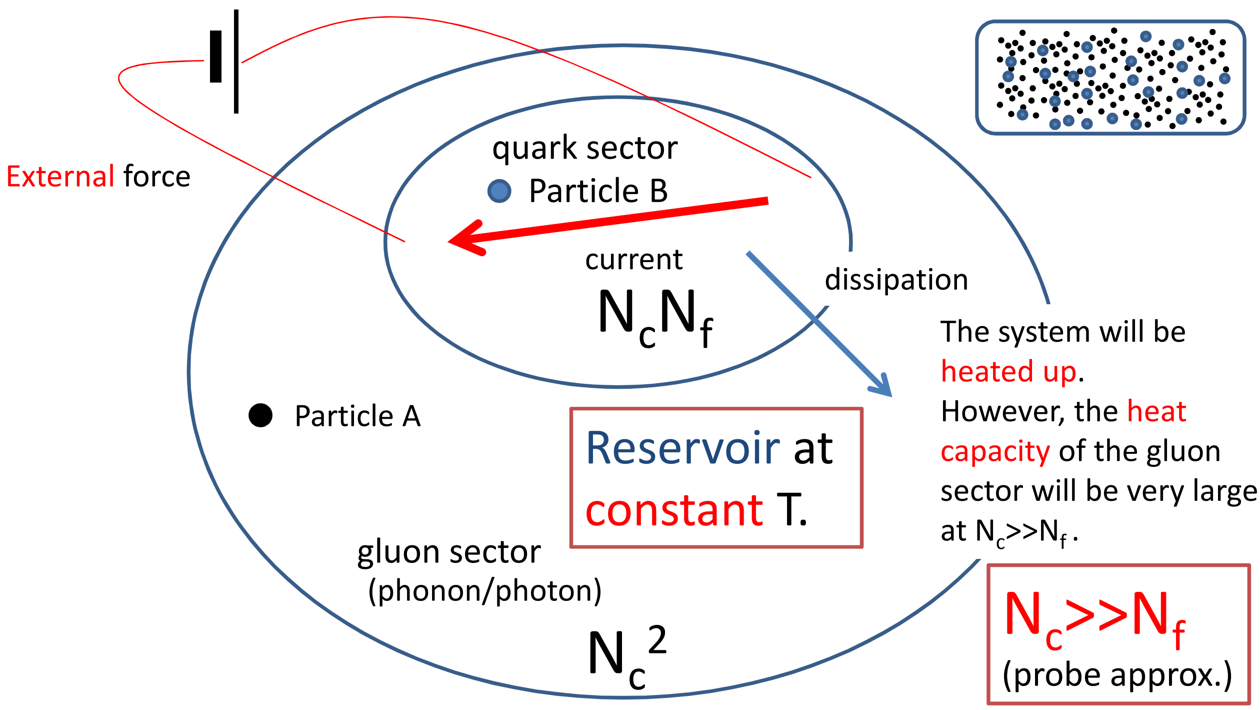
quarks
(and anti-quarks)

Gravity theory
(General relativity)

Map: AdS/CFT

J. Maldacena, 1997

How to realize the steady state with constant current in gauge theories?



We can consistently prepare the non-equilibrium steady state at the limit of $N_c \gg N_f$.

AdS/CFT correspondence

J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231.

Some categories of strongly-interacting quantum gauge theories are equivalent to higher-dimensional classical gravity theories.

What is the benefit of this correspondence?

- Strongly-interacting theory can be analyzed by using the weakly-interacting gravity.
- Quantum effects can be computed by using the classical gravity.

- Many-body physics is highly simplified in the gravity.

"Many-body physics" in the gravity

Particles A: gluons (heat bath) → single black hole

(Hawking and Bekenstein said that black hole has the notion of temperature and entropy.)

E. Witten, Adv. Theor. Math. Phys. 2 (1998) 505.

Particles B: quark/antiquarks → brane-like object (D-brane)

A. Karch and E. Katz, JHEP 0206 (2002) 043.

AdS/CFT is a tool to perform path integral of a given microscopic theory at given conditions, by solving the classical equations in gravity.

The gauge theory we employ

Since the computation of non-linear transport in non-equilibrium system is a challenge, let us employ the simplest example of the AdS/CFT correspondence.

The most standard example of AdS/CFT

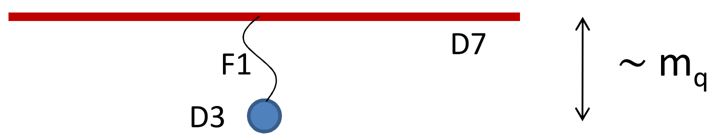
$\boxed{\text{N=4 SYM}} \Longleftrightarrow \boxed{AdS_5 \times S^5}$

However, all the fields in N=4 SYM are adjoint; there is no fundamental quark in this theory.

D3-D7 system

We can add the flavor degree of freedom (quarks and anti-quarks) by adding the D7-brane to the system.

(Karch and Katz, JHEP0206(2002)043)



The F1 string between the D3 and the D7 acts as a quark (or antiquark) from the viewpoint of the D3-branes.

$\boxed{\text{The gauge theory realized on the D3-branes is N=4 SYM + N=2 hyper-multiplet}}$

AdS/CFT based on D3-D7

SU(Nc) N=4 Supersymmetric Yang-Mills (SYM) theory at large-Nc with $\lambda = g_{YM}{}^2 Nc >> 1$.
（Quantum field theory） Finite T

+ quark sector of N_f falvors (N=2 hyper-multiplets)

Equivalent

Type IIBsupergravity at the classical level on weakly curved AdS-BH $\times S^5$

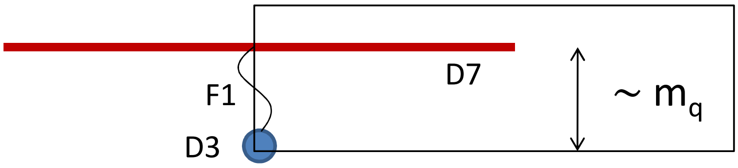
+ N_f D7-branes on this curved spacetime

You may say,

The “N=4 SYM + N=2 hypermultiplet” can never be a theory for realistic world.

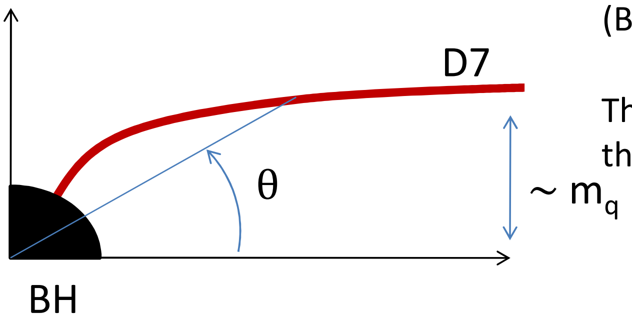
- But, our interest is not the microscopic details.
- The aim of the non-equilibrium statistical physics is to understand the macroscopic behavior of the systems that does not depend <u>too much</u> on the of microscopic details.

Gravity Dual



We draw only this part.

- The D3 is replaced with an AdS-BH in the gravity dual.



"z" represents the radial direction.
(But, the boundary is at z=0.)

The shape of the D7 is described by the function θ(z).

$$\theta(z) = m_q z + \mathrm{const.}\, z^3 + \ldots\ldots$$

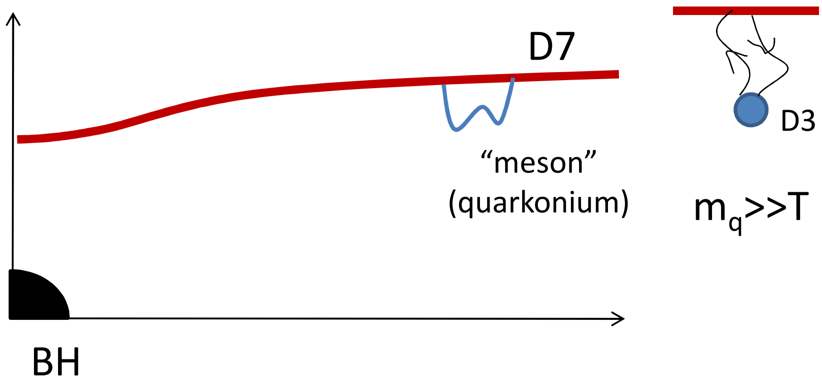
$$\left.\frac{1}{z}\sin\theta(z)\right|_{z\to 0} = m_q$$

Two phases for the quark sector

(The gluon sector is in the deconfinement phase since we have BH.)

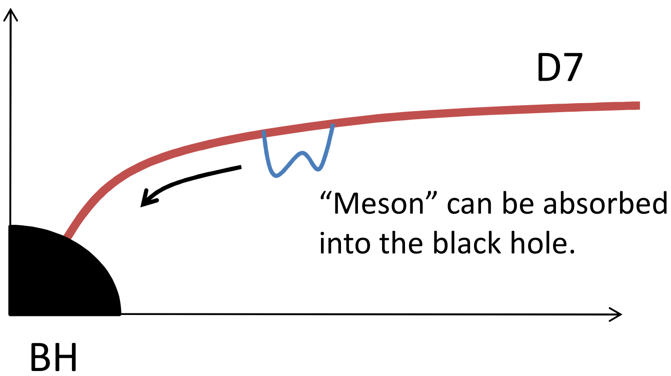
"Meson" is stable
in this phase.

sQGP-like phase



m_q>>T

"Meson" is unstable
in this phase.

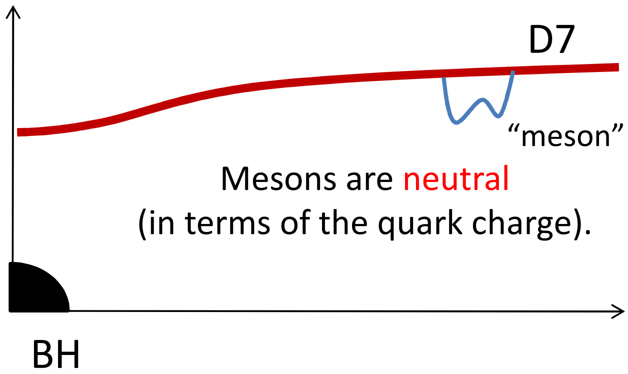


m_q<<T

From the viewpoint of condensed matter physics

"Mesons" are fundamental degree of freedom.

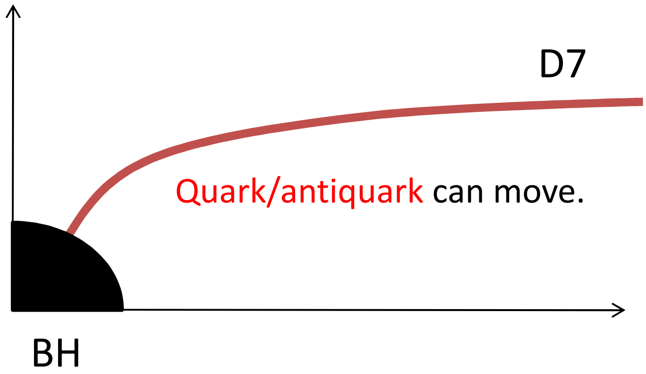
Insulator



$m_q >> T$

"Mesons" are unstable, and quarks/antiquarks are fundamental dof.

Conductor (metal)



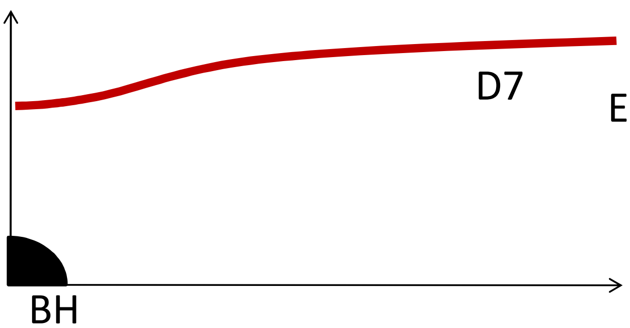
$m_q << T$

Breaking of insulation

(insulator phase)

(J. Erdmenger, R. Meyer, J.P. Shock, arXiv:0709.1551
T. Albash, V.F. Filev, C. Johnson, A. Kundu, arXiv:0709.1554)

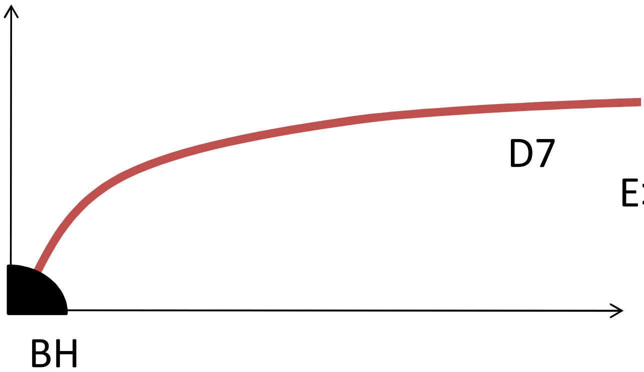
Insulator



$E < E_c(T, m_q)$

The external electric field (for quark charge) on the D7 changes the energy density of the D7 hence its shape.

Conductor (metal)



$E > E_c(T, m_q)$

Dirac-Born-Infeld (DBI) action for D7

$$S = -N_f T_{\mathrm{D7}} \int d^{7+1}x \sqrt{-\det G_{ab}}$$

$$G_{ab} = \partial_a x^\mu \partial_b x^\nu g_{\mu\nu} + (2\pi l_s^2) F_{ab}$$
$$+ \cancel{(\text{non-abelian part})}$$

We set this to be 1.

We consider only abelian part.

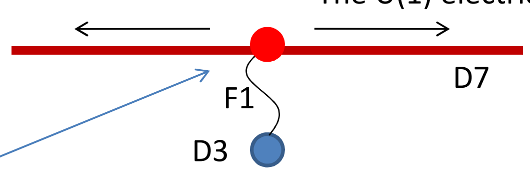
$$F_{ab} = \partial_a A_b - \partial_b A_a$$ The U(1) gauge field on the D7.

The non-trivial component of the induced metric is only the (z,z) component: $G_{zz}=1/z^2+\theta'(z)^2$.

The U(1) on the D7

The U(1) gauge field on the D7 is linked to the $U(1)_B$ charge ($U(1)_B$ current) in the YM side.

The U(1) electric field will be induced.



A_0 at the boundary will be lifted.

This endpoint acts as a unit charge of the U(1) gauge field on the D7.

If the quark moves (in x-direction), the "magnetic" field will be induced on the D7 and A_x at the boundary will be lifted as well.

AdS/CFT dictionary: GKP-Witten relation

z=0: boundary

$$A_0(z) = \mu - \frac{(2\pi)^2}{2N_c N_f}\left\langle J^0 \right\rangle z^2 + O(z^4)$$

$$A_x(z) = -Et + \frac{(2\pi)^2}{2N_c N_f}\left\langle J^x \right\rangle z^2 + O(z^4)$$

If the configuration of $A_x(z)$ on the D7-brane is specified as a function of z, the relationship between E and J^x can be read from it.

⇒ We obtain the (non-linear) conductivity.

However,

A_x obeys a second-order differential equation.

⇒ We need two boundary conditions to fix the solution.

The first and the second terms are the input conditions we need to specify by hand!

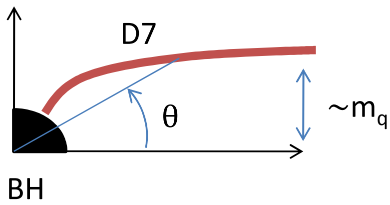
However, if we specify them as we like,
the on-shell D7-brane action will be complex in general.

The reality of the D7-brane action (the stability of the system) constrains the relationship between the first and the second terms.

[Karch, O'Bannon JHEP0709(2007)024]

The on-shell D7-brane action

$$S_{\mathrm{D7}} = -N\int dz dt \cos^6\theta \, g_{xx}^{5/2}\left|g_{tt}\right|^{1/2}\sqrt{W}$$



$$W = \frac{g_{zz}\left(\left|g_{tt}\right|g_{xx} - E^2\right)}{\left|g_{tt}\right|g_{xx}^3\cos^6\theta - \frac{g_{xx}\langle J_x\rangle^2}{N^2}}$$

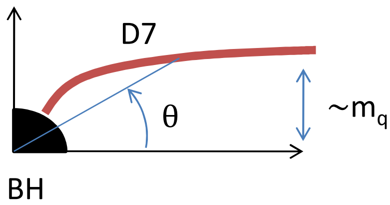
The metric of the AdS-BH

$$ds^2{}_{\mathrm{AdS\text{-}BH}} = -\frac{1}{z^2}\frac{\left(1 - z^4/z_H^4\right)^2}{1 + z^4/z_H^4}dt^2 + \frac{1 + z^4/z_H^4}{z^2}d\vec{x}^2 + \frac{dz^2}{z^2}$$

- The horizon is located at $z=z_H$.
- The boundary is at $z=0$.

On-shell D7-brane action

$$S_{\mathrm{D7}} = -N\int dzdt \cos^6\theta \, g_{xx}^{5/2} |g_{tt}|^{1/2} \sqrt{W}$$



$$W = \frac{g_{zz}\left(|g_{tt}|g_{xx} - E^2\right)}{|g_{tt}|g_{xx}^3 \cos^6\theta - \frac{g_{xx}\langle J_x\rangle^2}{N^2}}$$

Both the numerator and the denominator go across zero somewhere between the boundary and the horizon.

Only the way to make the action real is to make them go across zero at the same point.
(We define this point as z=z* .)

The conditions for reality

$$\left(-g_{tt}\right)g_{xx}\Big|_{z=z_*} - E^2 = 0 \quad \Rightarrow \quad z_* \text{ in terms of } E$$

$$\left(-g_{tt}\right)g_{xx}^3 \cos^6\theta - \frac{g_{xx}\langle J_x\rangle^2}{N^2}\Bigg|_{z=z_*} = 0$$

⇒ J_x is given in terms of E and θ(z_*).

$$\frac{1}{z}\sin\theta(z)\Big|_{z\to 0} = m_q$$

⇒ J_x is given by using E and m_q.

The conductivity is given as a function of m_q (at given T, λ).

The non-linear conductivity

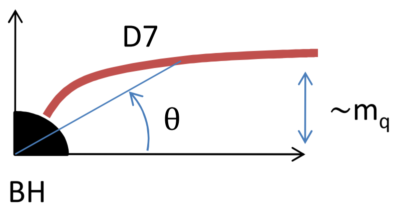
The charge density is also taken into account.
[Karch, O'Bannon JHEP0709(2007)024]

Normal conduction

$$\sigma_{xx} = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\cos^6\theta(z_*) + \frac{d^2}{e^2+1}}$$

Pair-creation

$$d \equiv \frac{\langle J^t \rangle}{\frac{\pi}{2}\sqrt{2\lambda}T^2}, \quad e \equiv \frac{E}{\frac{\pi}{2}\sqrt{2\lambda}T^2},$$



- $\cos\theta(z_*)$ goes to 1 at $m_q \to 0$.
- $\cos\theta(z_*)$ goes to zero at $m_q \to$ infinity.

Seems to be reasonable?

$$J_x\big|_{m_q\to\infty} = \frac{d}{\sqrt{1+e^2}}E \to \begin{cases} \approx & d\cdot E & (e << 1) \\ \approx & \text{saturate} & (e >> 1) \\ \approx & 0 & (T >> 1) \\ & (e \leftrightarrow -e \text{ symmetric}) & \end{cases}$$

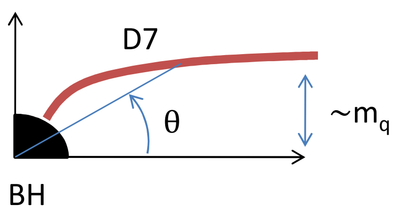
(Thanks to H. Hayakawa, H. Wada, A. Shimizu.)

The contribution of pair-creation

[Karch, O'Bannon JHEP0709(2007)024]

$$\sigma_{xx} = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\cos^6\theta(z_*)}$$

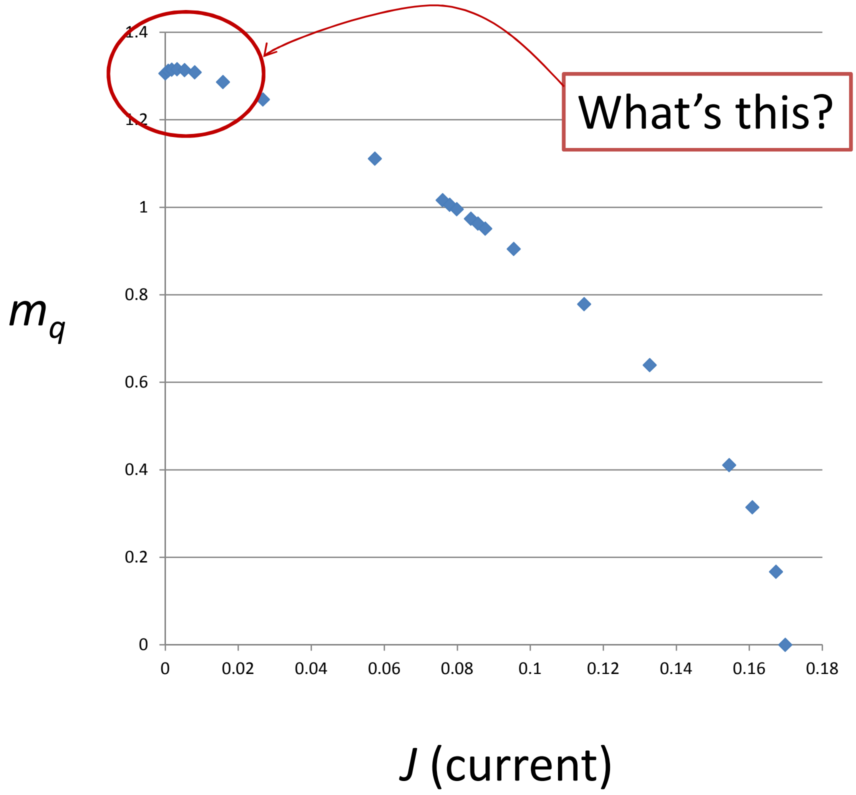
$$e \equiv \frac{E}{\frac{\pi}{2}\sqrt{2\lambda}T^2}$$



This function can be given by solving a non-linear differential equation, numerically.

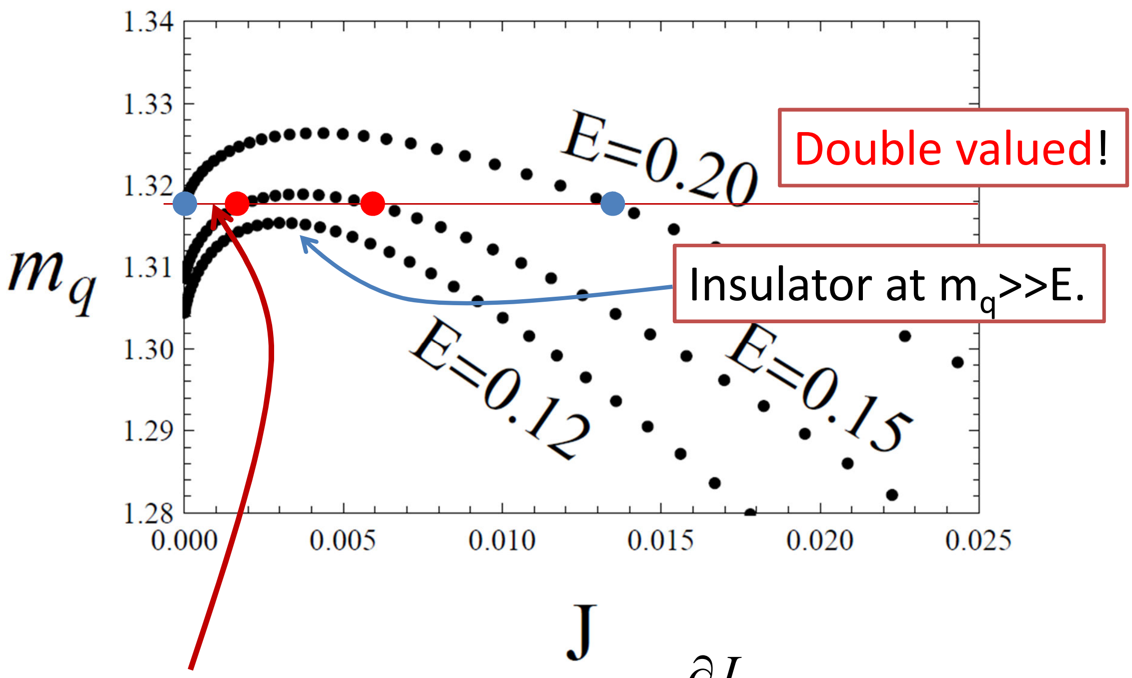
Let us explore the behavior of the conductivity.

J-m_q chracteristics



E=0.13 in our convention.

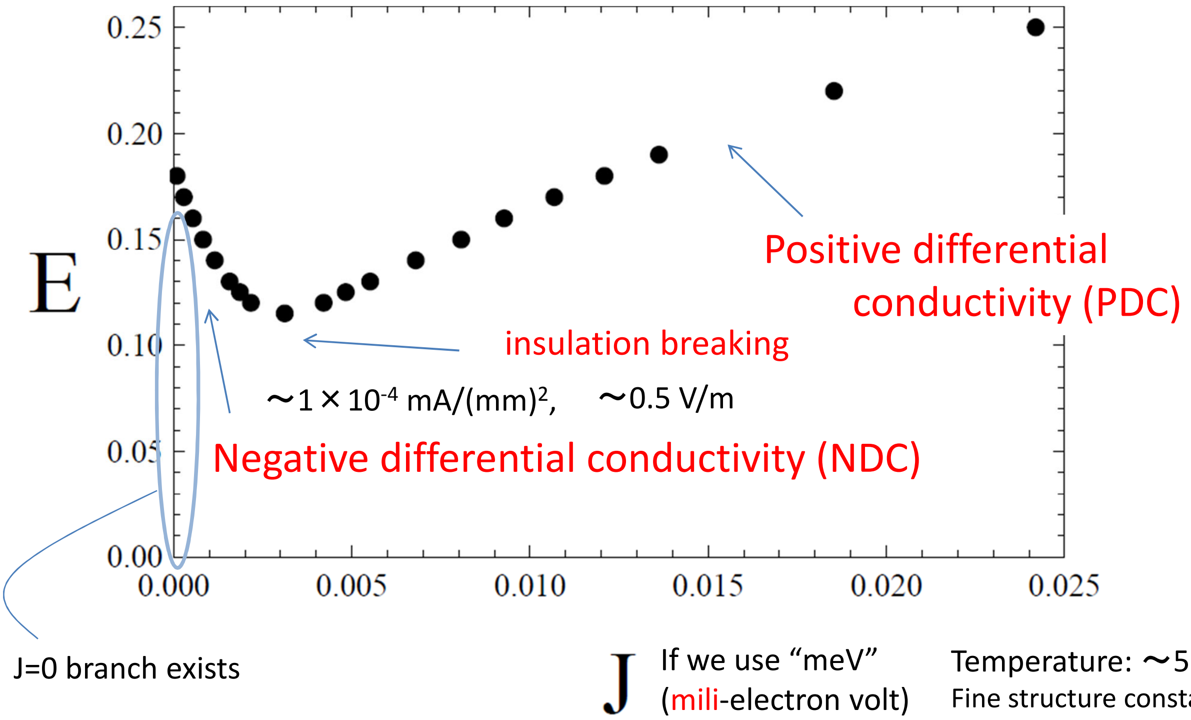
J-m_q chracteristics



Negative Differential Conductivity: $\frac{\partial J}{\partial E} < 0$

An example of J-E characteristics

S.N. Prog. Theor. Phys. 124(2010)1105.



If we use "meV" (mili-electron volt) as the unit.

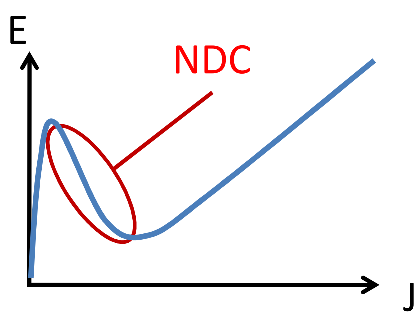
Temperature: $\sim$5 K
Fine structure constant read form the Coulomb interaction : $\sim$O(1)

m_q=1.315

Negative Differential Conductivity

Negative Differential Conductivity (NDC) has been widely observed in strongly-correlated systems of electrons.

(See, e.g. [Oka, Aoki, arXiv:0803.0422])

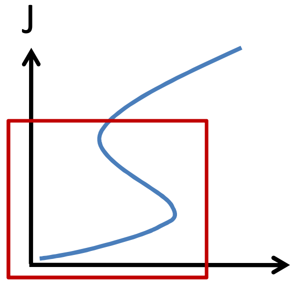


As far as I know, NDC of non-ballistic conduction in 3+1 dimensional systems has been reproduced from a microscopic theory for the first time.

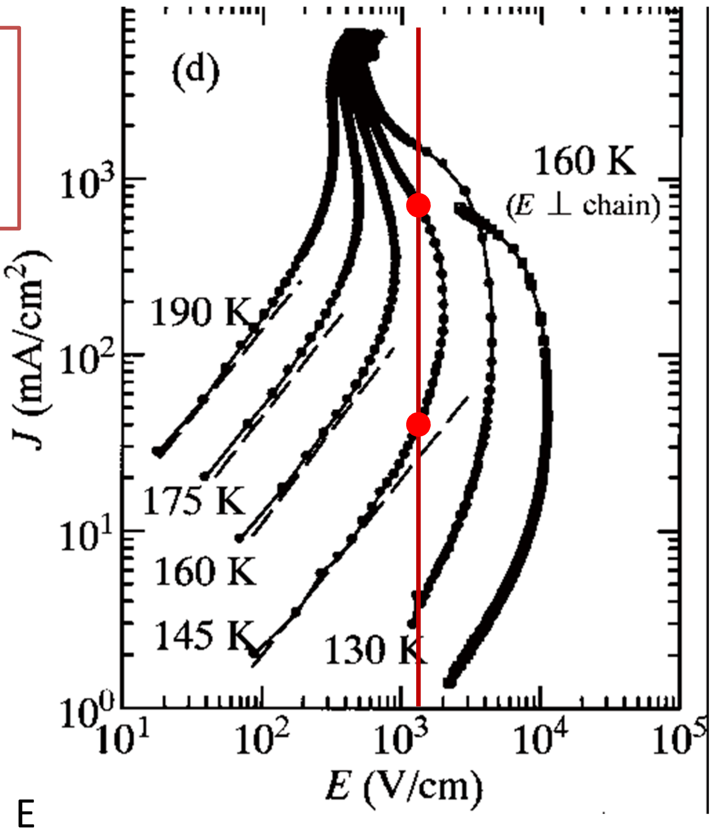
S.N. Prog. Theor. Phys. 124(2010)1105.

An example of experimental data.

$SrCuO_2$
(1d Mott)



We are looking here



Y. Taguchi T. Matsumoto and Y. Tokura. *Phys. Rev. B, 62:7015, 2000.*

Another experimental data

θ-(BEDT-TTF)$_2$CsCo(SCN)$_4$ crystal at 4.2 K.

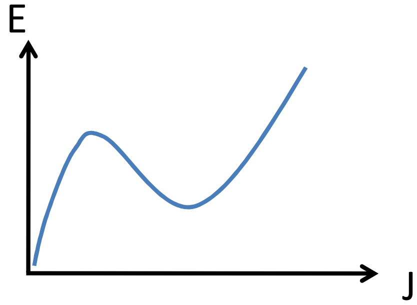
F. Sawano et. al., Nature 437 (2005) 522.

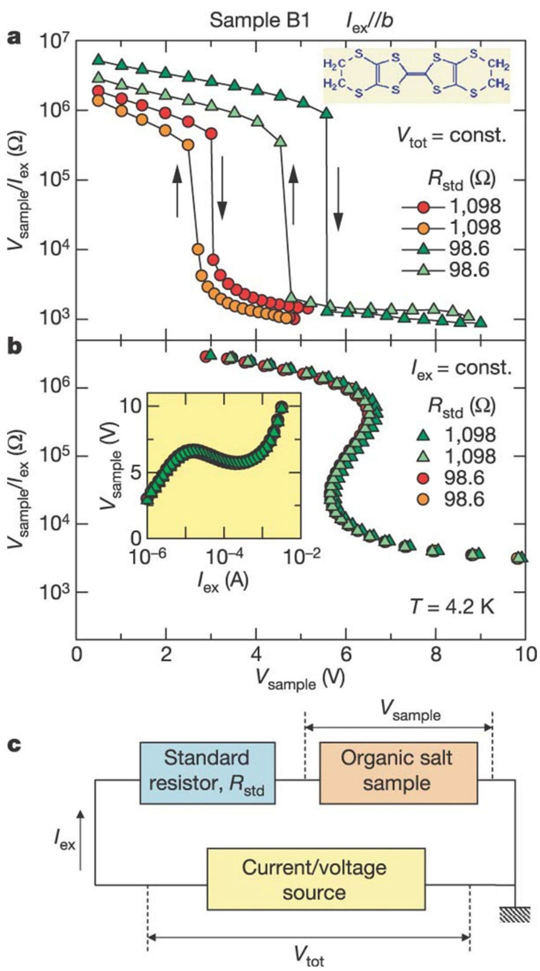
Charge order insulator

J(E) is a multi-valued function of E:

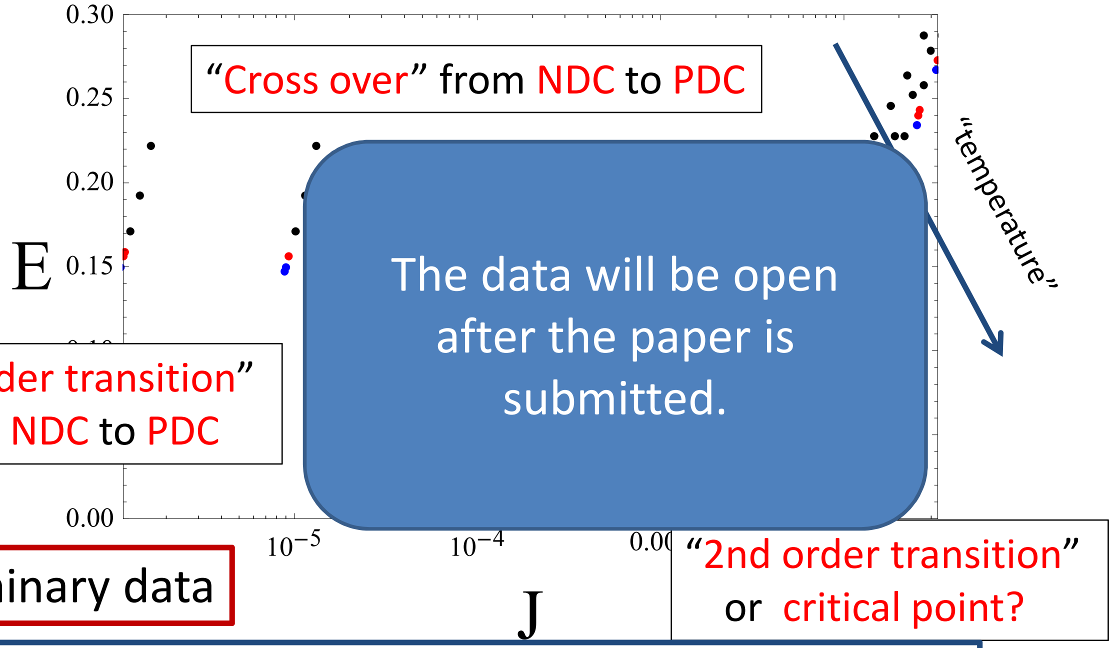
Experimental physicists measure E as a function of the controlled J.

E(J) is still a single-valued function.





Everything is in the nonlinear, non-equilibrium regime.



Preliminary data

First observation of the "this type" of phase transition!

Many questions arise

- Any critical phenomena at the "2nd-order" transition? Any massless mode?
- Any universality?
- Any critical exponents, scaling relations?
- Any Landau-like, or Van-der-Waals-like theory for the non-equilibrium transitions?
- Any observation?
- Any notion of the "effective temperature"?
- ....

Message

The AdS/CFT correspondence may open a new window for non-equilibrium physics.

Einstein came back to the non-equilibrium physics!

1905: Brownian motion, Relativity

100 years later → AdS/CFT

general relativity and black hole

I hope the cross-field application of AdS/CFT may open the physics of next 100 years.