On S-duality in non-Susy Gauge Theory

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(work in progress)

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1 Introduction

✩ What is S-duality?

▸ Maxwell theory

$$\partial_\mu F^{\mu\nu} = 0 \,, \quad \partial_\mu \tilde{F}^{\mu\nu} = 0 \qquad \left(\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \right)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,, \quad \tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$$

ele theory : field A_μ , coupling g_{ele}

mag theory : field $\tilde{A}_\mu$, coupling g_{mag}

→ equivalent! $g_{mag} = 1/g_{ele}$ (Dirac)

▸ $\mathcal{N}=4$ Super Yang-Mills Theory

Gauge group : G

field : A_μ , $\psi \times 4$, $\phi \times 6$ adjoint rep of G

gauge   Weyl fermion   scalar

<u>ele theory</u>   <u>mag theory</u>

dual

$G = SU(N) \leftrightarrow G = SU(N)$

$G = USp(2n) \leftrightarrow G = SO(2n+1)$ etc.

unitary symplectic gr.

again, $g_{mag} = 1/g_{ele}$

$USp(2n)$
$= \{ g \in U(2n) \mid g J g^T = J \}$

$J = \begin{pmatrix} & -1 \\ 1 & \end{pmatrix} \begin{matrix} \updownarrow n \\ \updownarrow n \end{matrix}$

- Type IIB string theory

↑

One of 10 dim superstring theory

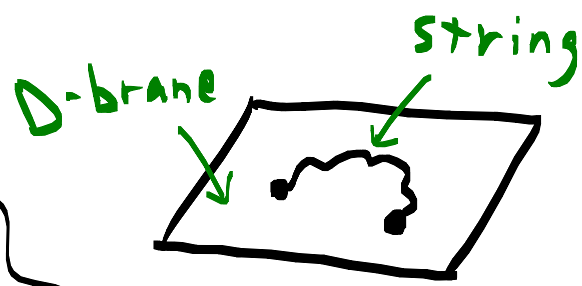
→ believed to be self-dual !

ele theory $\overset{\text{dual}}{\longleftrightarrow}$		mag theory
String	$\mapsto$	D1-brane
D1-brane	$\mapsto$	String
D3-brane	$\mapsto$	D3-brane
D5-brane	$\mapsto$	NS5-brane
NS5-brane	$\mapsto$	D5-brane

again, $g_{mag} = 1/g_{ele}$

Dp-brane

= (p+1) dim object, on which strings can end



★ Claim

S-duality in Type IIB theory

ele theory $\overset{\text{dual}}{\longleftrightarrow}$ **mag theory**

} low energy ↓   } low energy ↓

	gauge $USp(2n)$	global $SO(6)$
A_μ	⊟ (= adj.)	1
ψ	⊟	4
ϕ	⊟	6

	gauge $SO(2n-1)$	global $SO(6)$
$\tilde{A}_\mu$	⊟ (= adj.)	1
$\tilde{\psi}$	⊟	4
$\tilde{\phi}$	⊟	6

(Uranga '99)
↑
(mainly focused on brane dynamics)

- Strong at IR   - weak at IR

beleived to be

① confining ② DSB : global $SO(6) \simeq SU(4) \xrightarrow{\langle\psi\psi\rangle \neq 0} SO(4)$

Q. Can we understand ①, ② using string theory ?

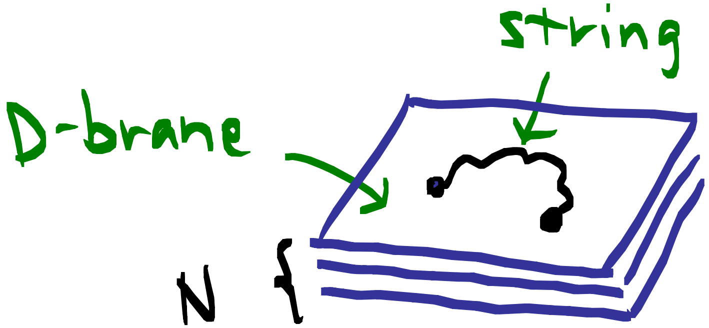
Plan

1. Introduction
2. D-brane & O-plane
3. $O3$-$\overline{D3}$ system
4. Confinement & DSB
5. Summary

② D-brane & O-plane

* What is D-brane ?

Dp-brane = (p+1) dim object,
on which strings can end



gauge particle
as massless mode

N Dp-branes $\Rightarrow$ (p+1)-dim
U(N) gauge theory
realized on the D-brane

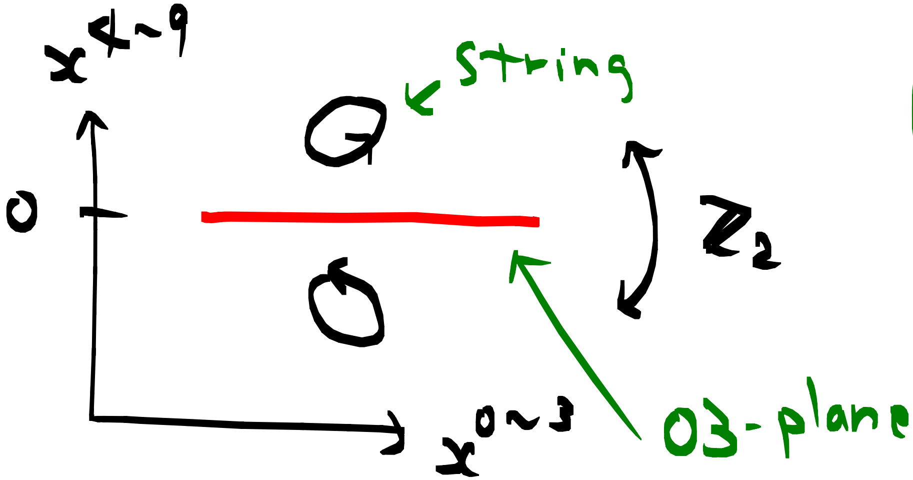
★ What is O3-plane?

- Consider $\mathbb{Z}_2$ action $\begin{cases} \cdot\ x^i \longrightarrow -x^i \\ \quad (i = 4, 5, \cdots, 9) \\ \cdot \text{ orientation of strings flipped} \end{cases}$

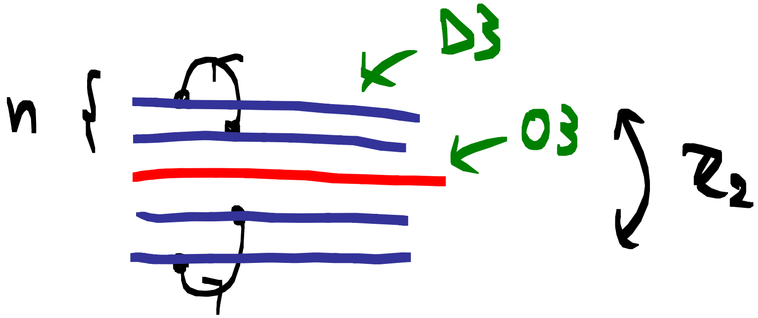
- $\left(\begin{matrix}\text{Type IIB} \\ \text{String theory}\end{matrix}\right) \Big/ \mathbb{Z}_2$ is consistent theory

- fixed plane of $\mathbb{Z}_2$ is called $\underline{\text{O3-plane}}$

$\uparrow$ (3+1) dim extended along $x^{0\sim3}$



★ $O3 + n\,D3$



There are several consistent types of $\mathbb{Z}_2$ projection

→ $O3^-$, $O3^+$, $\widetilde{O3}^-$, ...

$$\left(\begin{array}{lll} O3^- + n\,D3 & \to & \mathcal{N}=4\ SO(2n)\ \text{SYM} \\ O3^+ + n\,D3 & \to & \mathcal{N}=4\ USp(2n)\ \text{SYM} \\ \widetilde{O3}^- + n\,D3 & \to & \mathcal{N}=4\ SO(2n+1)\ \text{SYM} \end{array}\right.$$

$\mathcal{N}=4\ SO(2n)$ SYM: S-dual

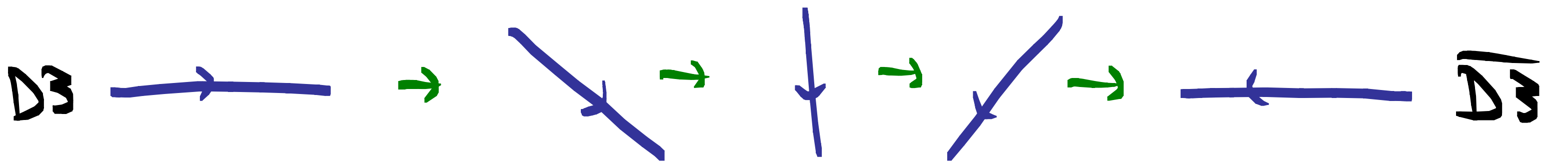
$\mathcal{N}=4\ USp(2n)$ SYM $\longleftrightarrow$ $\mathcal{N}=4\ SO(2n+1)$ SYM: S-dual

→ $\boxed{\widetilde{O3}^- \sim O3^- + \frac{1}{2}D3}$

- $O3^-$ S-dual
- $O3^+ \xleftrightarrow[\text{S-dual}]{} \widetilde{O3}^-$

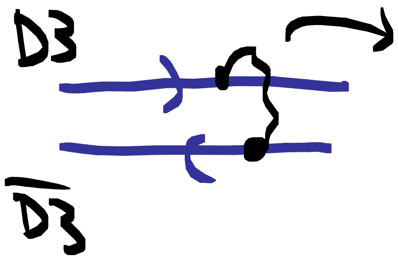
③ D3-$\overline{\text{D3}}$ system

★ What is $\overline{\text{D3}}$?



180° rotation

• D3-$\overline{\text{D3}}$ system



T : tachyon $\xrightarrow{\langle T \rangle \neq 0}$ nothing

Tachyon condensation $\longleftrightarrow$ D3-$\overline{\text{D3}}$ pair annihilation

(Sen '98)

• $O3^+ + n\,\overline{D3}$ → **ele**

	gauge $USp(2n)$	global $SO(6)$
A_μ	$\square\square$ (=adj.)	1
ψ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	4
ϕ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6

global $SO(6)$ ← rotation of $x^{4\sim 9}$-plane

(S.S. '99
Uranga '99)

s-dual

• $\widetilde{O3}^- + n\,\overline{D3} \simeq O3^- + \frac{1}{2}D3 + n\,\overline{D3} \underset{\langle T\rangle \neq 0}{\rightsquigarrow} O3^- + \frac{1}{2}(2n-1)\,\overline{D3}$

mag I

	gauge $SO(2n)$	global $SO(6)$
$\tilde{A}_\mu$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$ (=adj.)	1
$\tilde{\psi}$	$\square\square$	4
$\tilde{\phi}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6
T	$\square$	1
ψ'	$\square$	4^*

$\underset{\langle T\rangle \neq 0}{\rightsquigarrow}$

mag II

	gauge $SO(2n-1)$	global $SO(6)$
$\tilde{A}_\mu$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
$\tilde{\psi}$	$\square\square$	4
$\tilde{\phi}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6

④ Confinement & DSB

★ $n=1$ case

ele

	gauge $USp(2n)$	global $SO(6)$
A_μ	$\square\!\square$ (= adj.)	1
ψ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	4
ϕ	$\square\!\square$	6

$n=1$ ✓

$USp(2) \simeq SU(2)$

ψ : $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ is gauge singlet

ϕ : massive via quantum effect

$\Rightarrow$ $SU(2)$ pure YM

believed to be confining

mag I	gauge $SO(2n)$	global $SO(6)$
$\tilde{A}_\mu$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$ (=adj.)	1
$\tilde{\Psi}$	$\square\square$	4
$\tilde{\Phi}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6
T	$\square$	1
Ψ'	$\square$	4^*

$\xrightarrow{\langle T\rangle \neq 0}$

mag II	gauge $SO(2n-1)$	global $SO(6)$
$\tilde{A}_\mu$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
$\tilde{\Psi}$	$\square\square$	4
$\tilde{\Phi}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6

$\downarrow$ $n=1$

$SO(2) = U(1)^{mag}$

T is magnetic monopole

$\longrightarrow$

"monopole condensation"

$\downarrow$ $n=1$

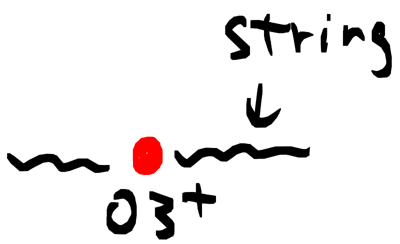
gauge sym is completely Higgsed

Confinement via dual Meissner effect !

(Nambu, 'tHooft, Mandelstam, Polyakov)

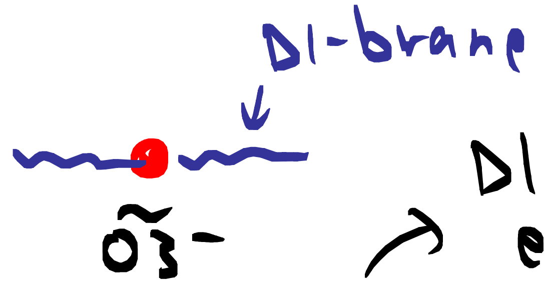
★ quark-anti-quark potential

Fact



Not allowed

dual $\longleftrightarrow$

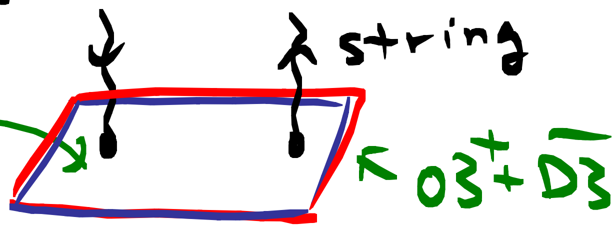


Not allowed $\nearrow$ D1 cannot end on $\left(\begin{array}{l} O3^- \\ \frac{1}{2} D3 \end{array} \right.$

ele : quark ~ string

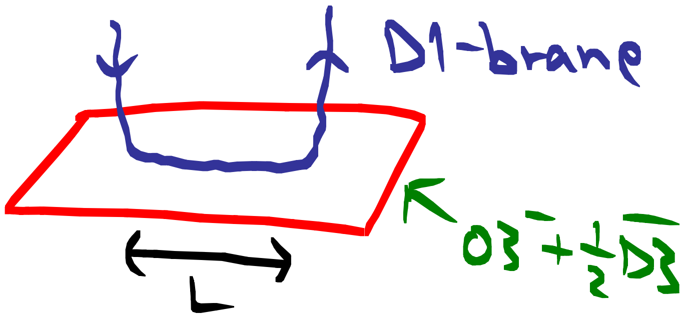
dual

ending on $\overline{D3}$



This is allowed at weak coupling

mag : D1 cannot end on $O3^- + \frac{1}{2}\overline{D3}$



$\Rightarrow \quad V(L) \propto L$

linear potential !

* $n > 1$ case

ele	gauge $USp(2n)$	global $SO(6)$
A_μ	$\square\!\square$ (= adj.)	$\mathbf{1}$
ψ	$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	$\mathbf{4}$
ϕ	$\square\!\square$	$\mathbf{6}$

- $\beta < 0$ weak at UV
- $m_\phi^2 \big|_{1\text{-loop}} > 0$

$O3^+$ $\overline{D3}$

attractive

@ We expect

$A, B = 1 \sim 4$: $\underline{4}$ of $SU(4) \simeq SO(6)$

$$\langle \psi^A_\alpha \psi^B_\beta \rangle \epsilon^{\alpha\beta} = \cancel{0}\, \delta^{AB}$$

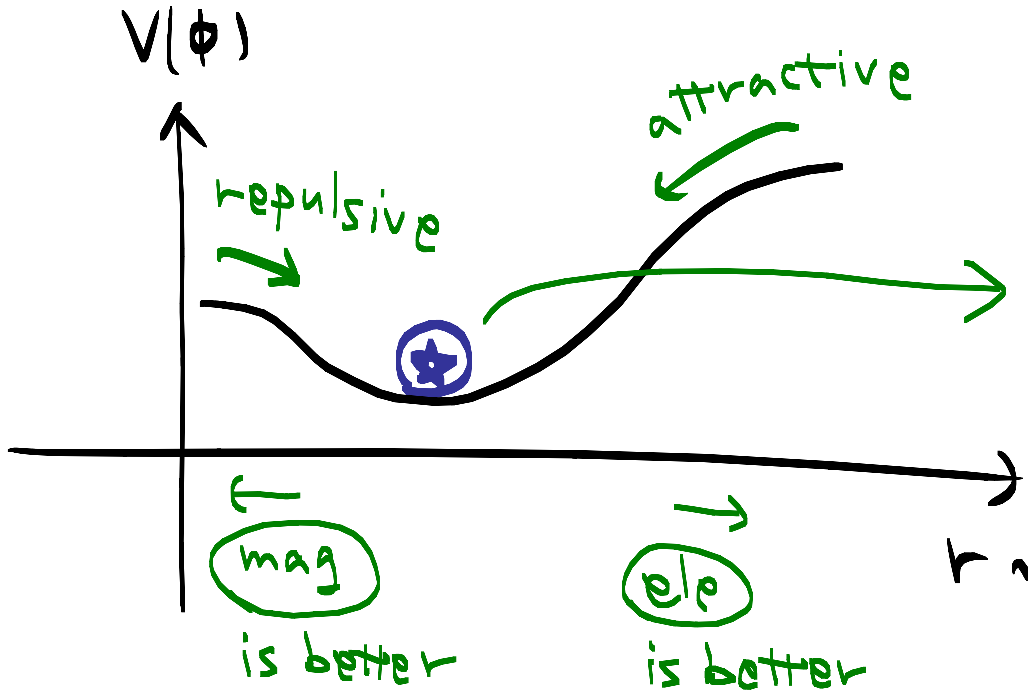
$$\Rightarrow \boxed{SO(6) \simeq SU(4) \xrightarrow[\text{broken to}]{} SO(4)}$$

mag II	gauge $SO(2n-1)$	global $SO(6)$
$\tilde{A}_\mu$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1
$\tilde{\Psi}$	$\square\square$	4
$\tilde{\phi}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6

- $\beta > 0$ weak at IR
- $m_\phi^2 \big|_{1\text{-loop}} < 0$

$O3^-$ $\overline{D3}$ $\rightsquigarrow$ repulsive

$\rightarrow$ We expect $\overline{D3}$ potential behaves like



$SO(6)$ is broken!

$r \sim \langle\phi\rangle \sim \langle\tilde{\phi}\rangle$

☆ speculative analysis (preliminary)

mag As a model, consider

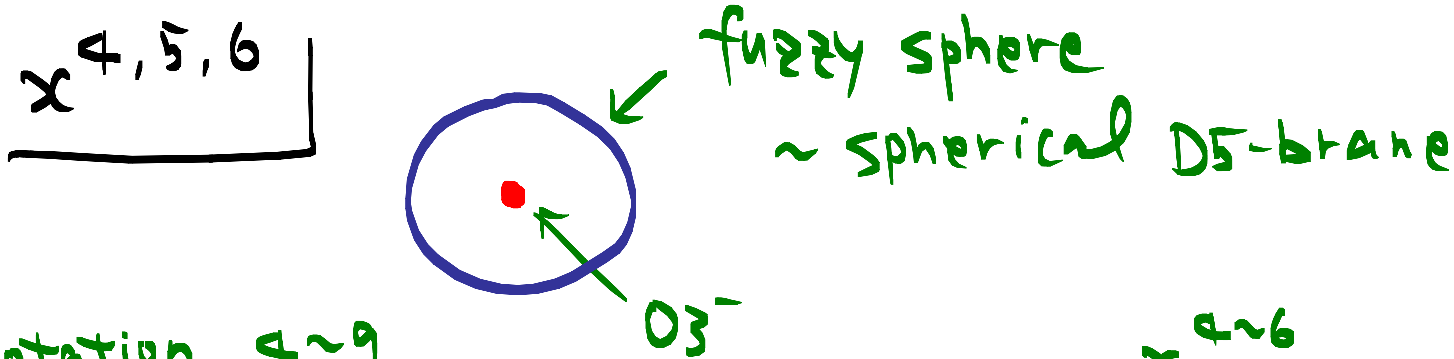
$$V(\tilde{\phi}) = -\underbrace{\frac{g}{4}}_{\text{tree level}}\mathrm{tr}\big([\overbrace{\tilde{\phi}^I}^{I=1\sim 6},\tilde{\phi}^J]^2\big) - \underbrace{\frac{\mu^2}{2}}_{\text{1-loop}}\mathrm{tr}\big(\tilde{\phi}^I\tilde{\phi}^I\big) + \overbrace{\frac{\lambda}{2}}^{\text{to stabilize}}\mathrm{tr}\big((\tilde{\phi}^I\tilde{\phi}^I)^2\big)$$

This model has a fuzzy sphere solution

$$\boxed{\tilde{\phi}^i = a J^i} \quad (i=1,2,3)$$

$$\left(\begin{array}{l} J^i : \text{spin } (n-1) \text{ rep. of } SU(2) \\ a = \sqrt{\dfrac{\mu^2}{2g+2\lambda n(n-1)}} \end{array} \right)$$

- This solution corresponds to



$\overset{\text{rotation of } x^{4\sim9}}{SO(6)}$ is broken to $\overset{x^{4\sim6}}{\underset{\simeq\, SU(2)}{SO(3)}} \times \overset{x^{7\sim9}}{\underset{\simeq\, SU(2)}{SO(3)}}$

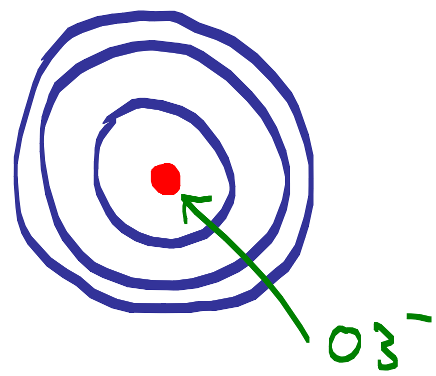
- This is consistent with DSB in ele !

$$SO(6) \simeq SU(4) \xrightarrow[\langle\psi\psi\rangle \neq 0]{} SO(4) \simeq SU(2)\times SU(2)$$

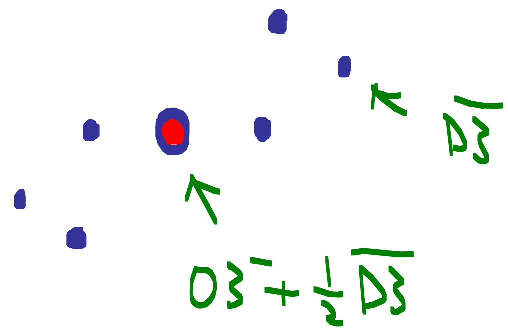
- $\overset{\text{gauge}}{SO(2n-1)}$ is completely broken $\longrightarrow$ Confinement in ele !

- <u>note</u>

$$\tilde{\Phi}^i = \begin{pmatrix} a_1 J_1^i & & & \\ & a_2 J_2^i & & \\ & & \ddots & \\ & & & a_k J_k^i \end{pmatrix}$$: multiple spheres



$$\tilde{\Phi}^i = \begin{pmatrix} v_1^i \sigma_2 & & & & \\ & v_2^i \sigma_2 & & & \\ & & \ddots & & \\ & & & v_n^i \sigma_2 & \\ & & & & 0 \end{pmatrix}$$: isolated $\overline{D3}$



$$\tilde{\Phi}^i = \begin{pmatrix} a_1 J_1^i & & & & \\ & \ddots & & & \\ & & a_k J_k^i & & \\ & & & v^i \sigma_2 & \\ & & & & \ddots \end{pmatrix}$$: spheres + isolated $\overline{D3}$

are higher energy config. provided $\lambda > g$.

⑤ Summary

* We have investigated non-SUSY S-duality based on $O3$-$\overline{D3}$ systems

* Useful to consider confinement and dynamical sym. breaking.